Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.7-d-trig-^m-a+b-c-cos-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [98]. This is test number [95].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (98)	% 0.00 (0)
Mathematica	% 100.00 (98)	% 0.00 (0)
Maple	% 100.00 (98)	% 0.00 (0)
Maxima	% 71.43 (70)	% 28.57 (28)
Fricas	% 74.49 (73)	% 25.51 (25)
Sympy	% 18.37 (18)	% 81.63 (80)
Giac	% 76.53 (75)	% 23.47 (23)
Mupad	% 68.37 (67)	% 31.63 (31)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

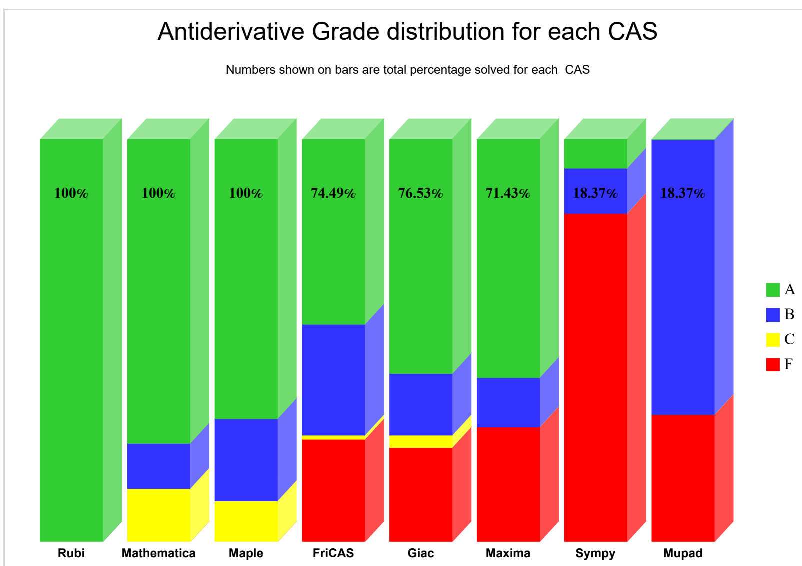
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

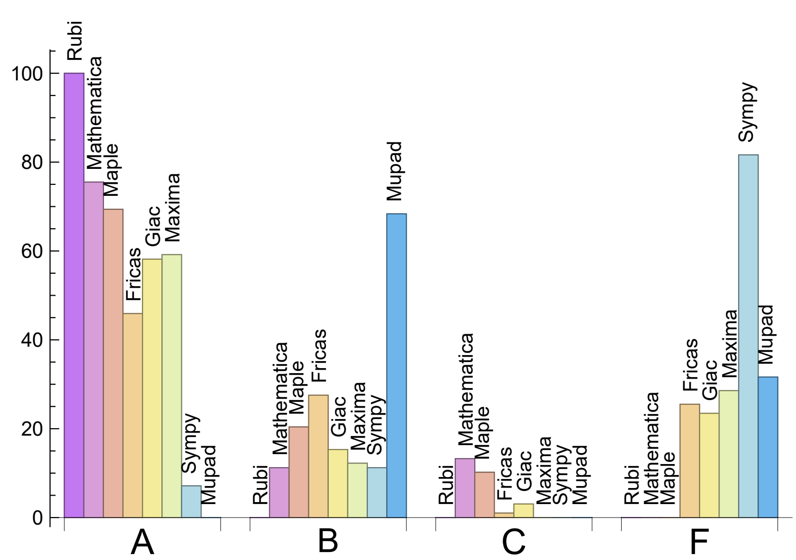
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.51	11.22	13.27	0.00
Maple	69.39	20.41	10.20	0.00
Maxima	59.18	12.24	0.00	28.57
Fricas	45.92	27.55	1.02	25.51
Sympy	7.14	11.22	0.00	81.63
Giac	58.16	15.31	3.06	23.47
Mupad	0.00	68.37	0.00	31.63

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	28	100.00 %	0.00 %	0.00 %
Fricas	25	48.00 %	32.00 %	20.00 %
Sympy	80	66.25 %	33.75 %	0.00 %
Giac	23	91.30 %	8.70 %	0.00 %
Mupad	31	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

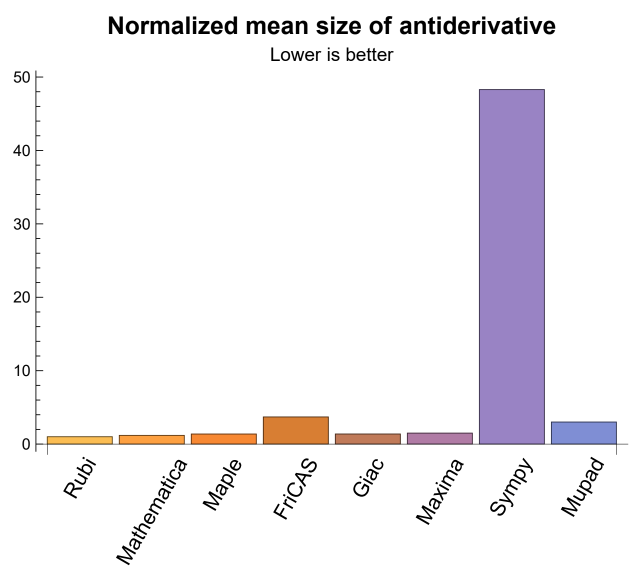
1.3 Performance

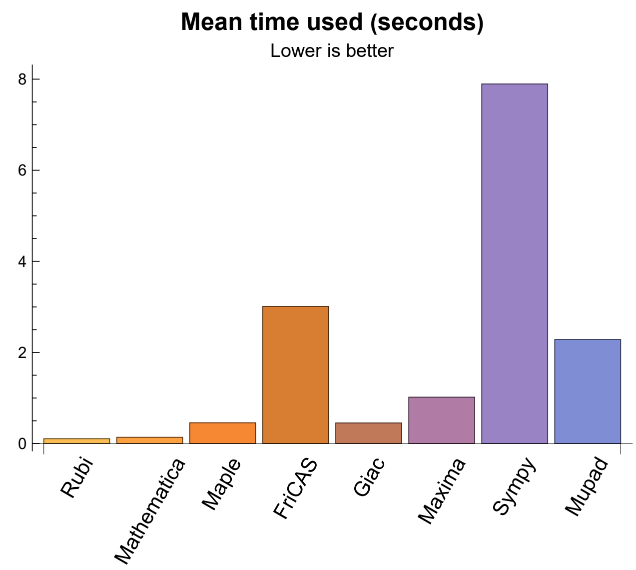
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	71.99	1.00	41.50	1.00
Mathematica	0.14	67.76	1.19	45.00	1.00
Maple	0.46	97.43	1.37	49.50	1.07
Maxima	1.02	58.53	1.49	41.50	1.09
Fricas	3.01	277.18	3.70	95.00	2.73
Sympy	7.89	1477.17	48.29	81.50	3.42
Giac	0.45	68.07	1.37	47.00	1.23
Mupad	2.28	274.70	3.00	75.00	1.06

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {74, 76, 77, 79}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

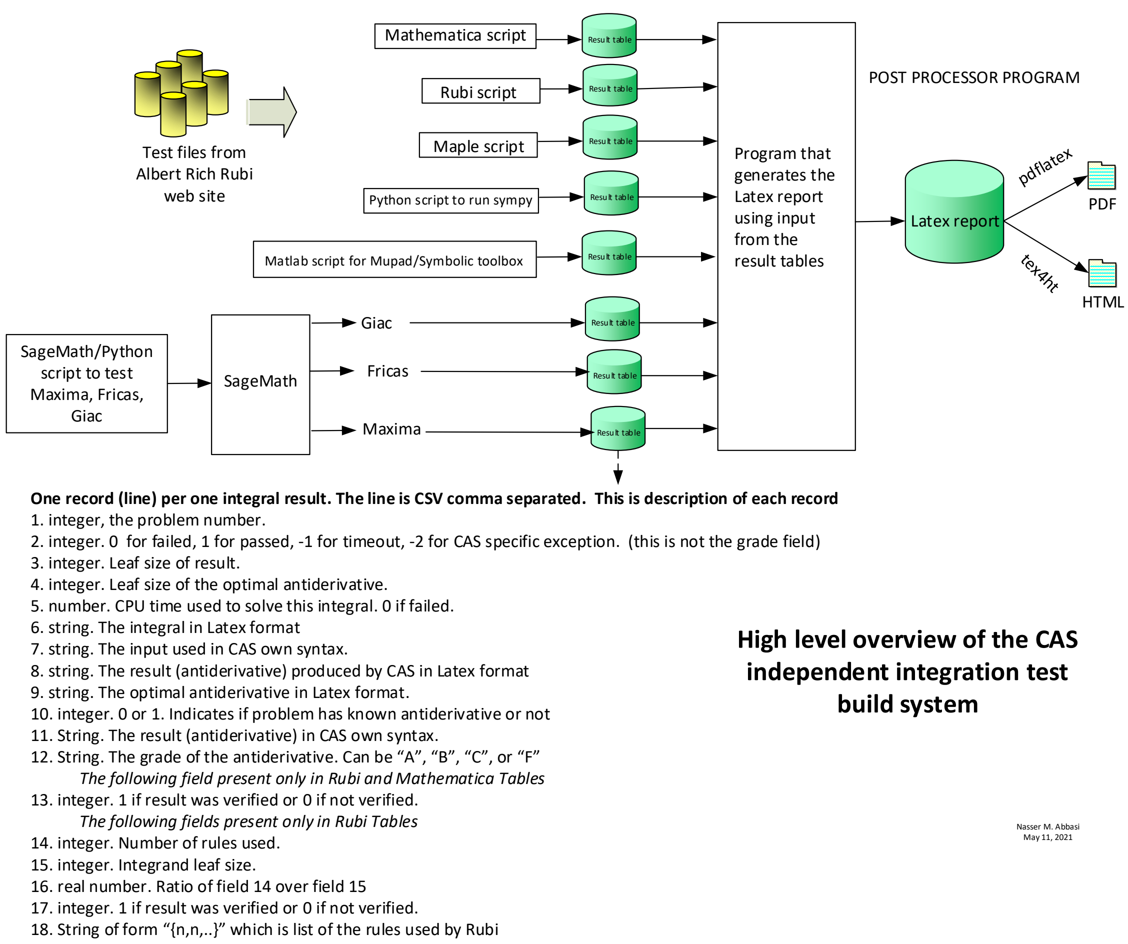
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 10, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 73, 81, 85, 86, 87, 89, 90, 93, 94, 95, 96, 97, 98 }

B grade: { 6, 7, 9, 11, 12, 15, 16, 33, 34, 88, 91 }

C grade: { 70, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 92 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 8, 11, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 57, 59, 60, 62, 63, 65, 66, 71, 72, 73, 81, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 7, 9, 10, 15, 16, 17, 18, 34, 52, 55, 56, 58, 61, 64, 67, 68, 69, 70, 84, 85 }

C grade: { 5, 74, 75, 76, 77, 78, 79, 80, 82, 83 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 67, 68, 69, 73, 81, 86, 92, 93, 94, 95, 97, 98 }

B grade: { 6, 7, 15, 16, 51, 53, 54, 87, 88, 89, 90, 91 }

C grade: { }

F grade: { 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 96 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 17, 18, 19, 24, 25, 28, 29, 30, 32, 33, 34, 35, 36, 37, 44, 45, 46, 47, 48, 49, 50, 51, 53, 73, 86, 87, 88, 89, 90, 93, 95, 96, 97, 98 }

B grade: { 6, 7, 9, 15, 16, 20, 21, 22, 23, 26, 27, 31, 38, 39, 40, 41, 42, 43, 67, 68, 69, 70, 71, 72, 81, 85, 91 }

C grade: { 92 }

F grade: { 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 94 }

2.1.6 Sympy

A grade: { 5, 13, 20, 24, 38, 44, 73 }

B grade: { 1, 2, 3, 4, 6, 25, 26, 27, 45, 46, 84 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 67, 68, 69, 70, 72, 73, 86, 87, 89, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 6, 7, 15, 16, 23, 47, 49, 53, 71, 81, 84, 85, 88, 90, 91 }

C grade: { 48, 50, 54 }

F grade: { 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 80, 82, 83 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 55, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92 }

C grade: { }

F grade: { 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	42	37	23	473	31	25
normalized size	1	1.00	0.79	1.27	1.12	0.70	14.33	0.94	0.76
time (sec)	N/A	0.052	0.005	0.080	0.854	0.673	6.362	0.179	2.114

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	14	14	78	14	16
normalized size	1	1.00	1.00	0.84	0.74	0.74	4.11	0.74	0.84
time (sec)	N/A	0.045	0.003	0.069	0.574	1.085	3.745	0.198	2.035

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	25	21	14	153	22	15
normalized size	1	1.00	0.90	1.25	1.05	0.70	7.65	1.10	0.75
time (sec)	N/A	0.045	0.003	0.078	0.862	1.137	2.268	0.173	2.037

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	12	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	1.71	1.00	1.00
time (sec)	N/A	0.041	0.002	0.062	0.659	0.633	1.237	0.158	0.024

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	2	5	5
normalized size	1	1.00	1.00	1.60	1.00	1.00	0.40	1.00	1.00
time (sec)	N/A	0.040	0.001	0.086	0.884	0.626	0.707	0.217	2.050

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	21	9	21	22	19	23	8
normalized size	1	1.00	2.62	1.12	2.62	2.75	2.38	2.88	1.00
time (sec)	N/A	0.025	0.006	0.052	0.312	0.629	0.224	0.156	2.112

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	44	37	48	0	38	26
normalized size	1	1.00	2.32	2.00	1.68	2.18	0.00	1.73	1.18
time (sec)	N/A	0.044	0.008	0.103	0.484	1.452	0.000	0.147	0.077

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	17	29	0	17	13
normalized size	1	1.00	1.11	0.95	0.89	1.53	0.00	0.89	0.68
time (sec)	N/A	0.048	0.004	0.109	0.321	0.465	0.000	0.317	2.029

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	75	66	51	72	0	47	39
normalized size	1	1.00	2.14	1.89	1.46	2.06	0.00	1.34	1.11
time (sec)	N/A	0.055	0.007	0.120	0.703	0.583	0.000	0.825	0.082

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	143	138	87	225	0	99	100
normalized size	1	1.00	1.83	1.77	1.12	2.88	0.00	1.27	1.28
time (sec)	N/A	0.090	0.256	0.076	0.881	0.712	0.000	0.152	2.132

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	116	86	53	152	0	59	65
normalized size	1	1.00	2.15	1.59	0.98	2.81	0.00	1.09	1.20
time (sec)	N/A	0.073	0.182	0.066	0.826	1.298	0.000	0.166	0.097

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	46	30	95	0	30	28
normalized size	1	1.00	2.50	1.28	0.83	2.64	0.00	0.83	0.78
time (sec)	N/A	0.053	0.181	0.064	2.343	0.548	0.000	0.177	0.085

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	73	87	17	18
normalized size	1	1.00	1.00	0.69	0.65	2.81	3.35	0.65	0.69
time (sec)	N/A	0.027	0.021	0.043	0.923	0.739	1.067	0.182	2.227

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	50	56	48	113	0	50	853
normalized size	1	1.00	1.19	1.33	1.14	2.69	0.00	1.19	20.31
time (sec)	N/A	0.049	0.048	0.102	1.720	0.745	0.000	0.220	2.666

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	140	111	105	274	0	103	1138
normalized size	1	1.00	2.26	1.79	1.69	4.42	0.00	1.66	18.35
time (sec)	N/A	0.086	0.550	0.121	1.183	0.666	0.000	0.175	2.589

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	204	205	200	592	0	178	833
normalized size	1	1.00	2.17	2.18	2.13	6.30	0.00	1.89	8.86
time (sec)	N/A	0.138	1.466	0.126	0.726	0.871	0.000	0.194	5.277

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	194	112	285	0	119	681
normalized size	1	1.00	0.88	2.20	1.27	3.24	0.00	1.35	7.74
time (sec)	N/A	0.181	0.196	0.087	1.026	2.474	0.000	1.614	2.683

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	105	62	211	0	80	126
normalized size	1	1.00	0.87	1.75	1.03	3.52	0.00	1.33	2.10
time (sec)	N/A	0.105	0.103	0.075	0.772	0.616	0.000	0.190	2.453

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	54	33	177	0	50	108
normalized size	1	1.00	0.92	1.35	0.82	4.42	0.00	1.25	2.70
time (sec)	N/A	0.059	0.072	0.072	1.735	0.746	0.000	0.193	2.370

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	12026	37	24
normalized size	1	1.00	0.97	0.70	0.67	5.43	400.87	1.23	0.80
time (sec)	N/A	0.019	0.059	0.056	0.953	0.577	38.278	0.162	2.384

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	39	38	228	0	55	34
normalized size	1	1.00	0.98	0.95	0.93	5.56	0.00	1.34	0.83
time (sec)	N/A	0.056	0.093	0.111	0.739	0.670	0.000	0.343	2.302

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	65	70	396	0	90	67
normalized size	1	1.00	0.97	1.07	1.15	6.49	0.00	1.48	1.10
time (sec)	N/A	0.080	0.215	0.116	1.092	0.626	0.000	0.869	2.335

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	106	127	610	0	156	101
normalized size	1	1.00	1.01	1.19	1.43	6.85	0.00	1.75	1.13
time (sec)	N/A	0.102	0.382	0.126	0.847	0.864	0.000	0.599	2.367

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	79	80	89	71	85	60	75
normalized size	1	1.00	0.81	0.82	0.91	0.72	0.87	0.61	0.77
time (sec)	N/A	0.090	0.091	0.053	0.917	0.572	1.649	0.198	0.307

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	6	8	14	6	4
normalized size	1	1.00	1.00	1.75	1.50	2.00	3.50	1.50	1.00
time (sec)	N/A	0.014	0.003	0.064	0.498	0.441	0.429	0.269	2.244

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	14	14	25	34	14	10
normalized size	1	1.00	1.31	1.08	1.08	1.92	2.62	1.08	0.77
time (sec)	N/A	0.015	0.004	0.072	0.866	1.004	1.137	0.168	2.249

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	20	20	37	54	20	17
normalized size	1	1.00	1.29	0.95	0.95	1.76	2.57	0.95	0.81
time (sec)	N/A	0.017	0.004	0.071	0.306	0.582	3.010	0.587	2.237

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	111	78	91	259	0	96	86
normalized size	1	1.00	1.42	1.00	1.17	3.32	0.00	1.23	1.10
time (sec)	N/A	0.084	0.422	0.065	1.081	2.131	0.000	0.180	0.134

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	86	50	67	191	0	65	51
normalized size	1	1.00	1.54	0.89	1.20	3.41	0.00	1.16	0.91
time (sec)	N/A	0.071	0.196	0.058	0.693	0.652	0.000	0.162	2.312

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	50	134	0	41	30
normalized size	1	1.00	1.00	0.87	1.32	3.53	0.00	1.08	0.79
time (sec)	N/A	0.055	0.033	0.059	0.543	0.629	0.000	0.176	0.101

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	21	39	95	0	31	21
normalized size	1	1.00	1.00	0.72	1.34	3.28	0.00	1.07	0.72
time (sec)	N/A	0.031	0.012	0.046	0.850	0.562	0.000	0.182	0.091

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	47	64	119	0	57	414
normalized size	1	1.00	0.93	1.15	1.56	2.90	0.00	1.39	10.10
time (sec)	N/A	0.052	0.057	0.096	0.924	0.596	0.000	0.187	2.500

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	152	92	92	186	0	85	483
normalized size	1	1.00	2.58	1.56	1.56	3.15	0.00	1.44	8.19
time (sec)	N/A	0.098	0.388	0.116	1.261	0.674	0.000	0.191	2.527

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	215	165	145	270	0	127	969
normalized size	1	1.00	2.39	1.83	1.61	3.00	0.00	1.41	10.77
time (sec)	N/A	0.166	1.184	0.126	1.994	0.937	0.000	0.194	2.639

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	76	122	97	273	0	104	1036
normalized size	1	1.00	0.87	1.40	1.11	3.14	0.00	1.20	11.91
time (sec)	N/A	0.196	0.227	0.070	1.049	1.015	0.000	0.187	2.694

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	60	54	213	0	72	291
normalized size	1	1.00	0.87	1.00	0.90	3.55	0.00	1.20	4.85
time (sec)	N/A	0.098	0.127	0.071	0.910	0.698	0.000	0.219	2.612

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	32	31	183	0	48	425
normalized size	1	1.00	0.95	0.84	0.82	4.82	0.00	1.26	11.18
time (sec)	N/A	0.060	0.085	0.061	1.395	0.623	0.000	1.176	2.549

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	12026	37	24
normalized size	1	1.00	0.97	0.70	0.67	5.43	400.87	1.23	0.80
time (sec)	N/A	0.019	0.052	0.053	1.431	0.501	38.680	0.376	0.002

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	33	32	216	0	36	30
normalized size	1	1.00	1.03	0.89	0.86	5.84	0.00	0.97	0.81
time (sec)	N/A	0.061	0.083	0.099	0.888	0.734	0.000	0.242	2.379

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	51	48	276	0	71	51
normalized size	1	1.00	0.98	0.91	0.86	4.93	0.00	1.27	0.91
time (sec)	N/A	0.086	0.150	0.109	0.870	0.950	0.000	0.166	2.309

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	80	74	348	0	104	84
normalized size	1	1.00	1.01	1.01	0.94	4.41	0.00	1.32	1.06
time (sec)	N/A	0.101	0.347	0.108	0.862	1.201	0.000	0.182	2.304

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	81	72	326	0	69	52
normalized size	1	1.00	1.08	1.25	1.11	5.02	0.00	1.06	0.80
time (sec)	N/A	0.049	0.243	0.069	1.090	0.744	0.000	0.178	2.341

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	106	175	186	616	0	149	123
normalized size	1	1.00	0.99	1.64	1.74	5.76	0.00	1.39	1.15
time (sec)	N/A	0.117	0.698	0.070	1.132	0.765	0.000	0.159	2.440

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	15	14	13	31	63	46	26
normalized size	1	1.00	0.44	0.41	0.38	0.91	1.85	1.35	0.76
time (sec)	N/A	0.013	0.025	0.052	0.820	2.582	0.666	0.192	2.325

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	27	26	57	218	59	40
normalized size	1	1.00	0.64	0.49	0.47	1.04	3.96	1.07	0.73
time (sec)	N/A	0.026	0.079	0.059	0.960	0.619	3.444	1.295	2.170

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	51	35	41	81	439	68	53
normalized size	1	1.00	0.72	0.49	0.58	1.14	6.18	0.96	0.75
time (sec)	N/A	0.052	0.129	0.060	1.569	1.754	13.610	0.145	2.149

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	10	4	0	24	10
normalized size	1	1.00	1.00	1.08	0.83	0.33	0.00	2.00	0.83
time (sec)	N/A	0.016	0.007	0.863	0.886	0.689	0.000	1.710	0.035

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	12	1	0	28	39
normalized size	1	1.00	1.00	1.00	0.86	0.07	0.00	2.00	2.79
time (sec)	N/A	0.016	0.006	0.535	0.877	1.466	0.000	1.247	2.295

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	19	11	11	0	45	-1
normalized size	1	1.00	0.79	0.66	0.38	0.38	0.00	1.55	-0.03
time (sec)	N/A	0.022	0.027	1.432	0.977	0.680	0.000	0.228	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	0	1	0	55	-1
normalized size	1	1.00	0.76	0.64	0.00	0.03	0.00	1.67	-0.03
time (sec)	N/A	0.025	0.028	0.990	0.000	0.447	0.000	0.647	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	28	14	35	19	0	0	-1
normalized size	1	1.00	1.87	0.93	2.33	1.27	0.00	0.00	-0.07
time (sec)	N/A	0.020	0.017	0.709	1.129	1.349	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	34	17	0	0	0	-1
normalized size	1	1.00	1.76	2.00	1.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.018	0.010	0.412	0.900	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	51	37	300	44	0	78	-1
normalized size	1	1.00	1.59	1.16	9.38	1.38	0.00	2.44	-0.03
time (sec)	N/A	0.027	0.050	1.565	1.835	0.603	0.000	0.256	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	53	51	284	0	0	90	-1
normalized size	1	1.00	1.47	1.42	7.89	0.00	0.00	2.50	-0.03
time (sec)	N/A	0.026	0.033	1.176	1.053	0.000	0.000	0.686	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	0	0	0	7
normalized size	1	1.00	1.22	4.56	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.008	0.023	1.556	0.000	0.626	0.000	0.000	0.009

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	75	0	0	0	0	-1
normalized size	1	1.00	1.06	2.34	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.039	2.405	0.000	0.829	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	49	0	0	0	0	-1
normalized size	1	1.00	1.10	1.17	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.078	0.974	0.000	0.593	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	101	0	0	0	0	-1
normalized size	1	1.00	0.91	2.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.052	2.339	0.000	0.574	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	110	0	0	0	0	-1
normalized size	1	1.00	0.74	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.073	2.622	0.000	0.630	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	123	192	0	0	0	0	-1
normalized size	1	1.00	1.02	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.481	1.793	0.000	0.512	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	0	0	0	-1
normalized size	1	1.00	1.22	4.56	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.008	0.035	1.813	0.000	0.470	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	62	0	0	0	0	-1
normalized size	1	1.00	1.03	1.94	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.018	0.040	2.138	0.000	0.509	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	48	0	0	0	0	-1
normalized size	1	1.00	1.10	1.14	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.071	0.695	0.000	0.531	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	35	70	0	0	0	0	-1
normalized size	1	1.00	1.09	2.19	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.018	0.063	2.338	0.000	0.681	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	101	0	0	0	0	-1
normalized size	1	1.00	0.77	1.80	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.044	2.820	0.000	0.580	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	75	73	0	0	0	0	-1
normalized size	1	1.00	0.96	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.185	1.851	0.000	0.569	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	33	8	49	0	8	-1
normalized size	1	1.00	1.00	3.67	0.89	5.44	0.00	0.89	-0.11
time (sec)	N/A	0.021	0.008	0.875	1.608	0.628	0.000	0.234	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	57	11	89	0	11	-1
normalized size	1	1.00	1.00	3.80	0.73	5.93	0.00	0.73	-0.07
time (sec)	N/A	0.027	0.026	1.043	0.699	1.390	0.000	0.424	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	53	8	39	0	16	-1
normalized size	1	1.00	1.00	5.89	0.89	4.33	0.00	1.78	-0.11
time (sec)	N/A	0.026	0.010	2.153	1.741	1.211	0.000	1.422	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	121	3350	0	809	0	307	926
normalized size	1	1.00	0.25	6.88	0.00	1.66	0.00	0.63	1.90
time (sec)	N/A	1.101	0.242	0.227	0.000	0.861	0.000	1.025	2.655

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	109	64	0	817	0	299	938
normalized size	1	1.00	1.08	0.63	0.00	8.09	0.00	2.96	9.29
time (sec)	N/A	0.116	0.198	0.089	0.000	0.713	0.000	1.127	2.596

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	45	227	0	3830	0	170	214
normalized size	1	1.00	0.15	0.78	0.00	13.12	0.00	0.58	0.73
time (sec)	N/A	0.191	0.077	0.216	0.000	42.423	0.000	0.665	2.730

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	24	21	20	43	78	53	20
normalized size	1	1.00	0.53	0.47	0.44	0.96	1.73	1.18	0.44
time (sec)	N/A	0.018	0.058	0.081	1.639	0.681	1.985	0.559	2.164

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	130	150	0	0	0	0	1520
normalized size	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	3.08
time (sec)	N/A	0.918	0.249	0.099	0.000	0.000	0.000	0.000	8.818

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	60	0	0	0	0	184
normalized size	1	1.00	0.85	0.35	0.00	0.00	0.00	0.00	1.08
time (sec)	N/A	0.238	0.228	0.464	0.000	0.000	0.000	0.000	3.081

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	172	76	0	0	0	0	216
normalized size	1	1.00	0.70	0.31	0.00	0.00	0.00	0.00	0.88
time (sec)	N/A	0.486	0.274	0.218	0.000	0.000	0.000	0.000	3.421

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	130	148	0	0	0	0	1518
normalized size	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	3.07
time (sec)	N/A	0.589	0.185	0.095	0.000	0.000	0.000	0.000	7.832

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	146	62	0	0	0	0	184
normalized size	1	1.00	0.83	0.35	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.249	0.180	0.429	0.000	0.000	0.000	0.000	3.120

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	78	0	0	0	0	216
normalized size	1	1.00	0.81	0.37	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.216	0.224	0.221	0.000	0.000	0.000	0.000	3.416

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	378	62	0	0	0	0	535
normalized size	1	1.00	1.70	0.28	0.00	0.00	0.00	0.00	2.40
time (sec)	N/A	0.562	0.127	0.071	0.000	0.000	0.000	0.000	2.778

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	103	79	73	72	138	0	185	99
normalized size	1	1.24	0.95	0.88	0.87	1.66	0.00	2.23	1.19
time (sec)	N/A	0.104	0.175	0.069	0.775	3.086	0.000	0.379	2.389

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	141	67	0	0	0	0	1025
normalized size	1	1.00	1.09	0.52	0.00	0.00	0.00	0.00	7.95
time (sec)	N/A	0.180	0.152	0.071	0.000	0.000	0.000	0.000	3.108

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	378	62	0	0	0	0	403
normalized size	1	1.00	1.84	0.30	0.00	0.00	0.00	0.00	1.97
time (sec)	N/A	0.473	0.122	0.076	0.000	0.000	0.000	0.000	2.447

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	117	233	0	0	728	199	95
normalized size	1	1.00	1.65	3.28	0.00	0.00	10.25	2.80	1.34
time (sec)	N/A	0.119	0.294	0.237	0.000	0.000	23.606	0.543	2.293

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	64	246	0	3963	0	222	241
normalized size	1	1.00	0.72	2.76	0.00	44.53	0.00	2.49	2.71
time (sec)	N/A	0.077	0.162	0.112	0.000	63.662	0.000	0.881	2.267

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	19	19	0	16	9
normalized size	1	1.00	1.00	0.94	1.12	1.12	0.00	0.94	0.53
time (sec)	N/A	0.029	0.009	0.101	0.303	0.528	0.000	0.454	2.163

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	43	95	90	0	38	-1
normalized size	1	1.00	1.00	1.08	2.38	2.25	0.00	0.95	-0.02
time (sec)	N/A	0.063	0.029	0.085	1.139	2.084	0.000	0.401	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	47	17	47	21	0	45	-1
normalized size	1	1.00	2.35	0.85	2.35	1.05	0.00	2.25	-0.05
time (sec)	N/A	0.051	0.037	0.987	0.959	0.497	0.000	0.370	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	81	67	0	24	-1
normalized size	1	1.00	1.00	1.20	3.24	2.68	0.00	0.96	-0.04
time (sec)	N/A	0.060	0.016	0.095	0.893	0.801	0.000	0.311	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	60	16	0	27	-1
normalized size	1	1.00	1.00	0.91	5.45	1.45	0.00	2.45	-0.09
time (sec)	N/A	0.035	0.009	0.087	0.884	0.675	0.000	0.366	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	44	8	39	17	0	33	-1
normalized size	1	1.00	4.89	0.89	4.33	1.89	0.00	3.67	-0.11
time (sec)	N/A	0.053	0.024	0.440	0.582	0.641	0.000	0.335	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	217	125	151	1690	0	143	1281
normalized size	1	1.00	1.42	0.82	0.99	11.05	0.00	0.93	8.37
time (sec)	N/A	0.197	0.301	0.109	1.593	46.276	0.000	2.134	5.211

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	52	123	0	38	-1
normalized size	1	1.00	1.00	0.76	1.16	2.73	0.00	0.84	-0.02
time (sec)	N/A	0.073	0.027	0.697	0.769	5.093	0.000	0.511	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	39	0	0	24	-1
normalized size	1	1.00	1.00	0.75	1.39	0.00	0.00	0.86	-0.04
time (sec)	N/A	0.067	0.017	0.125	1.760	0.000	0.000	0.409	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	44	43	90	0	38	-1
normalized size	1	1.00	1.00	0.98	0.96	2.00	0.00	0.84	-0.02
time (sec)	N/A	0.077	0.034	0.121	2.064	0.910	0.000	0.467	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	31	0	67	0	24	-1
normalized size	1	1.00	1.00	1.11	0.00	2.39	0.00	0.86	-0.04
time (sec)	N/A	0.071	0.018	0.122	0.000	0.910	0.000	0.423	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	39	58	97	0	46	-1
normalized size	1	1.00	0.98	0.83	1.23	2.06	0.00	0.98	-0.02
time (sec)	N/A	0.079	0.032	0.050	0.806	0.528	0.000	0.563	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	42	74	0	27	-1
normalized size	1	1.00	1.00	0.83	1.45	2.55	0.00	0.93	-0.03
time (sec)	N/A	0.077	0.017	0.061	1.039	0.535	0.000	0.560	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [72] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	16	0.188
2	A	3	2	1.00	16	0.125
3	A	3	3	1.00	16	0.188
4	A	2	2	1.00	16	0.125
5	A	2	2	1.00	16	0.125
6	A	2	2	1.00	14	0.143
7	A	3	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	3	2	1.00	16	0.125
9	A	4	3	1.00	16	0.188
10	A	4	3	1.00	15	0.200
11	A	4	3	1.00	15	0.200
12	A	3	3	1.00	15	0.200
13	A	2	2	1.00	13	0.154
14	A	4	4	1.00	13	0.308
15	A	5	5	1.00	15	0.333
16	A	6	6	1.00	15	0.400
17	A	6	6	1.00	15	0.400
18	A	5	5	1.00	15	0.333
19	A	4	4	1.00	15	0.267
20	A	2	2	1.00	10	0.200
21	A	3	3	1.00	15	0.200
22	A	4	3	1.00	15	0.200
23	A	4	3	1.00	15	0.200
24	A	7	7	1.00	13	0.538
25	A	3	3	1.00	10	0.300
26	A	3	2	1.00	10	0.200
27	A	3	2	1.00	10	0.200
28	A	4	3	1.00	15	0.200
29	A	4	3	1.00	15	0.200
30	A	3	3	1.00	15	0.200
31	A	2	2	1.00	13	0.154
32	A	4	4	1.00	13	0.308
33	A	5	5	1.00	15	0.333
34	A	6	6	1.00	15	0.400
35	A	6	6	1.00	15	0.400
36	A	5	5	1.00	15	0.333
37	A	3	3	1.00	15	0.200
38	A	2	2	1.00	10	0.200
39	A	3	3	1.00	15	0.200
40	A	4	3	1.00	15	0.200
41	A	4	3	1.00	15	0.200
42	A	4	4	1.00	10	0.400
43	A	5	5	1.00	10	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	2	2	1.00	8	0.250
45	A	4	4	1.00	8	0.500
46	A	5	5	1.00	8	0.625
47	A	3	3	1.00	12	0.250
48	A	3	3	1.00	10	0.300
49	A	4	4	1.00	12	0.333
50	A	4	4	1.00	10	0.400
51	A	3	3	1.00	12	0.250
52	A	3	3	1.00	10	0.300
53	A	4	4	1.00	12	0.333
54	A	4	4	1.00	10	0.400
55	A	1	1	1.00	10	0.100
56	A	2	2	1.00	12	0.167
57	A	2	2	1.00	12	0.167
58	A	4	4	1.00	10	0.400
59	A	6	6	1.00	12	0.500
60	A	6	6	1.00	12	0.500
61	A	1	1	1.00	10	0.100
62	A	2	2	1.00	12	0.167
63	A	2	2	1.00	12	0.167
64	A	3	3	1.00	10	0.300
65	A	4	4	1.00	12	0.333
66	A	4	4	1.00	12	0.333
67	A	2	2	1.00	13	0.154
68	A	2	2	1.00	21	0.095
69	A	2	2	1.00	15	0.133
70	A	10	6	1.00	10	0.600
71	A	4	3	1.00	11	0.273
72	A	10	6	1.00	8	0.750
73	A	3	3	1.00	10	0.300
74	A	12	3	1.00	10	0.300
75	A	7	3	1.00	10	0.300
76	A	9	3	1.00	10	0.300
77	A	12	3	1.00	11	0.273
78	A	7	3	1.00	11	0.273
79	A	9	3	1.00	11	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	11	5	1.00	8	0.625
81	A	7	3	1.24	8	0.375
82	A	9	3	1.00	8	0.375
83	A	11	5	1.00	10	0.500
84	A	8	6	1.00	10	0.600
85	A	10	6	1.00	10	0.600
86	A	4	4	1.00	11	0.364
87	A	4	4	1.00	15	0.267
88	A	5	5	1.00	15	0.333
89	A	3	3	1.00	15	0.200
90	A	3	3	1.00	13	0.231
91	A	4	4	1.00	15	0.267
92	A	11	10	1.00	15	0.667
93	A	5	5	1.00	15	0.333
94	A	4	4	1.00	15	0.267
95	A	5	5	1.00	15	0.333
96	A	4	4	1.00	15	0.267
97	A	5	5	1.00	15	0.333
98	A	4	4	1.00	15	0.267

Chapter 3

Listing of integrals

3.1 $\int \frac{\sin^6(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a}-\frac{\sin^3(x)\cos(x)}{4a}-\frac{3\sin(x)\cos(x)}{8a}$$

```
[Out] 3/8*x/a-3/8*cos(x)*sin(x)/a-1/4*cos(x)*sin(x)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{3x}{8a}-\frac{\sin^3(x)\cos(x)}{4a}-\frac{3\sin(x)\cos(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a - a*Cos[x]^2),x]
```

```
[Out] (3*x)/(8*a) - (3*Cos[x]*Sin[x])/(8*a) - (Cos[x]*Sin[x]^3)/(4*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(x)}{a - a\cos^2(x)}\,dx = \frac{\int \sin^4(x)\,dx}{a}$$
$$= -\frac{\cos(x)\sin^3(x)}{4a} + \frac{3\int \sin^2(x)\,dx}{4a}$$
$$= -\frac{3\cos(x)\sin(x)}{8a} - \frac{\cos(x)\sin^3(x)}{4a} + \frac{3\int 1\,dx}{8a}$$
$$= \frac{3x}{8a} - \frac{3\cos(x)\sin(x)}{8a} - \frac{\cos(x)\sin^3(x)}{4a}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a - a*Cos[x]^2),x]
[Out] ((3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32)/a
```

fricas [A] time = 0.67, size = 23, normalized size = 0.70

$$\frac{\left(2\cos(x)^3 - 5\cos(x)\right)\sin(x) + 3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] 1/8*((2*cos(x)^3 - 5*cos(x))*sin(x) + 3*x)/a
```

giac [A] time = 0.18, size = 31, normalized size = 0.94

$$\frac{3x}{8a} - \frac{5\tan(x)^3 + 3\tan(x)}{8\left(\tan(x)^2 + 1\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="giac")
[Out] 3/8*x/a - 1/8*(5*tan(x)^3 + 3*tan(x))/((tan(x)^2 + 1)^2*a)
```

maple [A] time = 0.08, size = 42, normalized size = 1.27

$$-\frac{5\left(\tan^3(x)\right)}{8a\left(\tan^2(x)+1\right)^2} - \frac{3\tan(x)}{8a\left(\tan^2(x)+1\right)^2} + \frac{3\arctan\left(\tan(x)\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a-a*cos(x)^2),x)
[Out] -5/8/a/(tan(x)^2+1)^2*tan(x)^3-3/8/a/(tan(x)^2+1)^2*tan(x)+3/8/a*arctan(tan
(x))
```

maxima [A] time = 0.85, size = 37, normalized size = 1.12

$$-\frac{5\tan(x)^3 + 3\tan(x)}{8\left(a\tan(x)^4 + 2a\tan(x)^2 + a\right)} + \frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(5*tan(x)^3 + 3*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

mupad [B] time = 2.11, size = 25, normalized size = 0.76

$$\frac{\sin(4x)}{32a} - \frac{\sin(2x)}{4a} + \frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a - a*cos(x)^2),x)
```

```
[Out] sin(4*x)/(32*a) - sin(2*x)/(4*a) + (3*x)/(8*a)
```

sympy [B] time = 6.36, size = 473, normalized size = 14.33

$$\frac{3x\tan^8\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right)+32a\tan^6\left(\frac{x}{2}\right)+48a\tan^4\left(\frac{x}{2}\right)+32a\tan^2\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^6\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right)+32a\tan^6\left(\frac{x}{2}\right)+48a\tan^4\left(\frac{x}{2}\right)+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a-a*cos(x)**2),x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**
6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*tan(
x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 1
2*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(
x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/2)**7/(8*a*tan(x/2)**8 + 32*a*t
an(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 22*tan(x/2)**5/(8
*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8
*a) - 22*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
 + 32*a*tan(x/2)**2 + 8*a) - 6*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6
 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

3.2 $\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\cos^3(x)}{3a}-\frac{\cos(x)}{a}$$

```
[Out] -cos(x)/a+1/3*cos(x)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\cos^3(x)}{3a}-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a - a*Cos[x]^2),x]
[Out] -(Cos[x]/a) + Cos[x]^3/(3*a)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \sin^3(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)}{a}\\ &= -\frac{\cos(x)}{a}+\frac{\cos^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a - a*Cos[x]^2),x]
[Out] ((-3*Cos[x])/4 + Cos[3*x]/12)/a
```

fricas [A] time = 1.08, size = 14, normalized size = 0.74

$$\frac{\cos(x)^3-3\,\cos(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^3 - 3*cos(x))/a
```

giac [A] time = 0.20, size = 14, normalized size = 0.74

$$\frac{\cos(x)^3 - 3\cos(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*(cos(x)^3 - 3*cos(x))/a
```

maple [A] time = 0.07, size = 16, normalized size = 0.84

$$\frac{\frac{(\cos^3(x))}{3} - \cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a-a*cos(x)^2),x)
```

```
[Out] 1/a*(1/3*cos(x)^3-cos(x))
```

maxima [A] time = 0.57, size = 14, normalized size = 0.74

$$\frac{\cos(x)^3 - 3\cos(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(cos(x)^3 - 3*cos(x))/a
```

mupad [B] time = 2.04, size = 16, normalized size = 0.84

$$-\frac{3\cos(x) - \cos(x)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a - a*cos(x)^2),x)
```

```
[Out] -(3*cos(x) - cos(x)^3)/(3*a)
```

sympy [B] time = 3.74, size = 78, normalized size = 4.11

$$-\frac{12\tan^2\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a} - \frac{4}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a-a*cos(x)**2),x)
```

```
[Out] -12*tan(x/2)**2/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
 - 4/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
```

3.3 $\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}-\frac{\sin(x)\cos(x)}{2a}$$

```
[Out] 1/2*x/a-1/2*cos(x)*sin(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{x}{2a}-\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a - a*Cos[x]^2),x]
```

```
[Out] x/(2*a) - (Cos[x]*Sin[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \sin^2(x)\,dx}{a}\\
&= -\frac{\cos(x)\sin(x)}{2a}+\frac{\int 1\,dx}{2a}\\
&= \frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}-\frac{1}{4}\sin(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a - a*Cos[x]^2),x]
```

```
[Out] (x/2 - Sin[2*x]/4)/a
```

fricas [A] time = 1.14, size = 14, normalized size = 0.70

$$-\frac{\cos(x)\sin(x) - x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*sin(x) - x)/a
```

giac [A] time = 0.17, size = 22, normalized size = 1.10

$$\frac{x}{2\,a} - \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*x/a - 1/2*tan(x)/((tan(x)^2 + 1)*a)
```

maple [A] time = 0.08, size = 25, normalized size = 1.25

$$-\frac{\tan(x)}{2a\left(\tan^2(x) + 1\right)} + \frac{\arctan\left(\tan(x)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a-a*cos(x)^2),x)
```

```
[Out] -1/2/a*tan(x)/(tan(x)^2+1)+1/2/a*arctan(tan(x))
```

maxima [A] time = 0.86, size = 21, normalized size = 1.05

$$\frac{x}{2\,a} - \frac{\tan(x)}{2\left(a\tan(x)^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/a - 1/2*tan(x)/(a*tan(x)^2 + a)
```

mupad [B] time = 2.04, size = 15, normalized size = 0.75

$$\frac{2\,x - \sin\left(2\,x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a - a*cos(x)^2),x)
```

```
[Out] (2*x - sin(2*x))/(4*a)
```

sympy [B] time = 2.27, size = 153, normalized size = 7.65

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2}{2a\tan^4\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a-a*cos(x)**2),x)
```

```
[Out] x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/(
2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x/2
)**2 + 2*a) + 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) - 2*t
an(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)
```

3.4 $\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=7

$$-\frac{\cos(x)}{a}$$

```
[Out] -cos(x)/a
```

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2638}

$$-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a - a*Cos[x]^2),x]
```

```
[Out] -(Cos[x]/a)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \sin(x)\,dx}{a}\\ &= -\frac{\cos(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a - a*Cos[x]^2),x]
```

```
[Out] -(Cos[x]/a)
```

fricas [A] time = 0.63, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -cos(x)/a
```

giac [A] time = 0.16, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="giac")
[Out] -cos(x)/a
```

maple [A] time = 0.06, size = 8, normalized size = 1.14

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a-a*cos(x)^2),x)
[Out] -cos(x)/a
```

maxima [A] time = 0.66, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] -cos(x)/a
```

mupad [B] time = 0.02, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a - a*cos(x)^2),x)
[Out] -cos(x)/a
```

sympy [B] time = 1.24, size = 12, normalized size = 1.71

$$-\frac{2}{a\tan^2\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a-a*cos(x)**2),x)
[Out] -2/(a*tan(x/2)**2 + a)
```

3.5 $\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

```
[Out] x/a
```

Rubi [A] time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a - a*Cos[x]^2),x]
[Out] x/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a} \end{aligned}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a - a*Cos[x]^2),x]
[Out] x/a
```

fricas [A] time = 0.63, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] x/a
```

giac [A] time = 0.22, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="giac")
[Out] x/a
```

maple [C] time = 0.09, size = 8, normalized size = 1.60

$$\frac{\arctan(\tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a-a*cos(x)^2),x)
[Out] 1/a*arctan(tan(x))
```

maxima [A] time = 0.88, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] x/a
```

mupad [B] time = 2.05, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a - a*cos(x)^2),x)
[Out] x/a
```

sympy [A] time = 0.71, size = 2, normalized size = 0.40

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a-a*cos(x)**2),x)
[Out] x/a
```

3.6 $\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-\frac{\tanh^{-1}(\cos(x))}{a}$$

[Out] -arctanh(cos(x))/a

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3175, 3770}

$$-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a - a*Cos[x]^2),x]

[Out] -(ArcTanh[Cos[x]]/a)

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}\end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.62

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a - a*Cos[x]^2),x]

[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]])/a

fricas [B] time = 0.63, size = 22, normalized size = 2.75

$$-\frac{\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] -1/2*(log(1/2*cos(x) + 1/2) - log(-1/2*cos(x) + 1/2))/a
```

giac [B] time = 0.16, size = 23, normalized size = 2.88

$$-\frac{\log\left(\cos(x)+1\right)}{2\,a}+\frac{\log\left(-\cos(x)+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="giac")
[Out] -1/2*log(cos(x) + 1)/a + 1/2*log(-cos(x) + 1)/a
```

maple [A] time = 0.05, size = 9, normalized size = 1.12

$$-\frac{\operatorname{arctanh}\left(\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a-a*cos(x)^2),x)
[Out] -arctanh(cos(x))/a
```

maxima [B] time = 0.31, size = 21, normalized size = 2.62

$$-\frac{\log\left(\cos(x)+1\right)}{2\,a}+\frac{\log\left(\cos(x)-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] -1/2*log(cos(x) + 1)/a + 1/2*log(cos(x) - 1)/a
```

mupad [B] time = 2.11, size = 8, normalized size = 1.00

$$-\frac{\operatorname{atanh}\left(\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a - a*cos(x)^2),x)
[Out] -atanh(cos(x))/a
```

sympy [B] time = 0.22, size = 19, normalized size = 2.38

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2a}-\frac{\log\left(\cos\left(x\right)+1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)**2),x)
[Out] log(cos(x) - 1)/(2*a) - log(cos(x) + 1)/(2*a)
```

3.7 $\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}$$

```
[Out] -1/2*arctanh(cos(x))/a-1/2*cot(x)*csc(x)/a
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$-\frac{\tanh^{-1}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a - a*Cos[x]^2),x]
```

```
[Out] -ArcTanh[Cos[x]]/(2*a) - (Cot[x]*Csc[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc^3(x)\,dx}{a}\\
&= -\frac{\cot(x)\csc(x)}{2a}+\frac{\int \csc(x)\,dx}{2a}\\
&= -\frac{\tanh^{-1}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 51, normalized size = 2.32

$$\frac{-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a - a*Cos[x]^2),x]
```

```
[Out] (-1/8*Csc[x/2]^2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8)/a
```

fricas [B] time = 1.45, size = 48, normalized size = 2.18

$$-\frac{\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4\left(a\cos(x)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(a*cos(x)^2 - a)
```

giac [B] time = 0.15, size = 38, normalized size = 1.73

$$-\frac{\log\left(\cos(x)+1\right)}{4\,a}+\frac{\log\left(-\cos(x)+1\right)}{4\,a}+\frac{\cos(x)}{2\left(\cos(x)^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2*cos(x)/((cos(x)^2 - 1
)*a)
```

maple [B] time = 0.10, size = 44, normalized size = 2.00

$$\frac{1}{4a\left(-1+\cos(x)\right)}+\frac{\ln\left(-1+\cos(x)\right)}{4a}+\frac{1}{4a\left(\cos(x)+1\right)}-\frac{\ln\left(\cos(x)+1\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a-a*cos(x)^2),x)
```

```
[Out] 1/4/a/(-1+cos(x))+1/4/a*ln(-1+cos(x))+1/4/a/(cos(x)+1)-1/4*ln(cos(x)+1)/a
```

maxima [B] time = 0.48, size = 37, normalized size = 1.68

$$\frac{\cos(x)}{2\left(a\cos(x)^2-a\right)}-\frac{\log\left(\cos(x)+1\right)}{4\,a}+\frac{\log\left(\cos(x)-1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*cos(x)/(a*cos(x)^2 - a) - 1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a
```

mupad [B] time = 0.08, size = 26, normalized size = 1.18

$$-\frac{\cos(x)}{2\left(a-a\cos(x)^2\right)}-\frac{\operatorname{atanh}\left(\cos(x)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a - a*cos(x)^2)),x)
```

```
[Out] - cos(x)/(2*(a - a*cos(x)^2)) - atanh(cos(x))/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc(x)}{\cos^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)**2),x)
```

```
[Out] -Integral(csc(x)/(cos(x)**2 - 1), x)/a
```

3.8 $\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\cot^3(x)}{3a}-\frac{\cot(x)}{a}$$

```
[Out] -cot(x)/a-1/3*cot(x)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$-\frac{\cot^3(x)}{3a}-\frac{\cot(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a - a*Cos[x]^2),x]
[Out] -(Cot[x]/a) - Cot[x]^3/(3*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc^4(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)dx, x, \cot(x)\right)}{a}\\ &= -\frac{\cot(x)}{a}-\frac{\cot^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.11

$$\frac{-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a - a*Cos[x]^2),x]
[Out] ((-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3)/a
```

fricas [A] time = 0.47, size = 29, normalized size = 1.53

$$-\frac{2\cos(x)^3-3\cos(x)}{3\left(a\cos(x)^2-a\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((a*cos(x)^2 - a)*sin(x))
```

giac [A] time = 0.32, size = 17, normalized size = 0.89

$$-\frac{3\,\tan(x)^2+1}{3\,a\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/(a*tan(x)^3)
```

maple [A] time = 0.11, size = 18, normalized size = 0.95

$$\frac{-\frac{1}{3\tan(x)^3}-\frac{1}{\tan(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a-a*cos(x)^2),x)
```

```
[Out] 1/a*(-1/3/tan(x)^3-1/tan(x))
```

maxima [A] time = 0.32, size = 17, normalized size = 0.89

$$-\frac{3\,\tan(x)^2+1}{3\,a\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/(a*tan(x)^3)
```

mupad [B] time = 2.03, size = 13, normalized size = 0.68

$$-\frac{\cot(x)\,\left(\cot(x)^2+3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a - a*cos(x)^2)),x)
```

```
[Out] -(cot(x)*(cot(x)^2 + 3))/(3*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^2(x)}{\cos^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a-a*cos(x)**2),x)
```

```
[Out] -Integral(csc(x)**2/(cos(x)**2 - 1), x)/a
```

3.9 $\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=35

$$-\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{\cot(x)\csc^3(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}$$

```
[Out] -3/8*arctanh(cos(x))/a-3/8*cot(x)*csc(x)/a-1/4*cot(x)*csc(x)^3/a
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 3768, 3770}

$$-\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{\cot(x)\csc^3(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a - a*Cos[x]^2),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(8*a) - (3*Cot[x]*Csc[x])/(8*a) - (Cot[x]*Csc[x]^3)/(4
*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc^5(x)\,dx}{a}\\ &= -\frac{\cot(x)\csc^3(x)}{4a}+\frac{3\int \csc^3(x)\,dx}{4a}\\ &= -\frac{3\cot(x)\csc(x)}{8a}-\frac{\cot(x)\csc^3(x)}{4a}+\frac{3\int \csc(x)\,dx}{8a}\\ &= -\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{3\cot(x)\csc(x)}{8a}-\frac{\cot(x)\csc^3(x)}{4a}\end{aligned}$$

Mathematica [B] time = 0.01, size = 75, normalized size = 2.14

$$\frac{-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{3}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{3}{32}\sec^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a - a*Cos[x]^2),x]
```

```
[Out] ((-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]
])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64)/a
```

fricas [B] time = 0.58, size = 72, normalized size = 2.06

$$\frac{6\cos(x)^3 - 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 1}{16\left(a\cos(x)^4 - 2a\cos(x)^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3*
(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(a*cos(x)^4
 - 2*a*cos(x)^2 + a)
```

giac [A] time = 0.83, size = 47, normalized size = 1.34

$$-\frac{3\log\left(\cos(x) + 1\right)}{16a} + \frac{3\log\left(-\cos(x) + 1\right)}{16a} + \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^2 - 1\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^3 - 5*cos
(x))/((cos(x)^2 - 1)^2*a)
```

maple [B] time = 0.12, size = 66, normalized size = 1.89

$$-\frac{1}{16a\left(-1+\cos(x)\right)^2} + \frac{3}{16a\left(-1+\cos(x)\right)} + \frac{3\ln\left(-1+\cos(x)\right)}{16a} + \frac{1}{16a\left(\cos(x)+1\right)^2} + \frac{3}{16a\left(\cos(x)+1\right)} - \frac{3\ln\left(\cos(x)+\right.}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a-a*cos(x)^2),x)
```

```
[Out] -1/16/a/(-1+cos(x))^2+3/16/a/(-1+cos(x))+3/16/a*ln(-1+cos(x))+1/16/a/(cos(x
)+1)^2+3/16/a/(cos(x)+1)-3/16*ln(cos(x)+1)/a
```

maxima [A] time = 0.70, size = 51, normalized size = 1.46

$$\frac{3\cos(x)^3 - 5\cos(x)}{8\left(a\cos(x)^4 - 2a\cos(x)^2 + a\right)} - \frac{3\log\left(\cos(x) + 1\right)}{16a} + \frac{3\log\left(\cos(x) - 1\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(a*cos(x)^4 - 2*a*cos(x)^2 + a) - 3/16*log(cos(
x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

mupad [B] time = 0.08, size = 39, normalized size = 1.11

$$-\frac{3\,\mathrm{atanh}\left(\cos(x)\right)}{8a} - \frac{\frac{5\cos(x)}{8} - \frac{3\cos(x)^3}{8}}{a\cos(x)^4 - 2a\cos(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a - a*cos(x)^2)),x)
```

```
[Out] - (3*atanh(cos(x)))/(8*a) - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(a - 2*a*cos(x)
^2 + a*cos(x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^3(x)}{\cos^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a-a*cos(x)**2),x)
```

```
[Out] -Integral(csc(x)**3/(cos(x)**2 - 1), x)/a
```

3.10 $\int \frac{\sin^7(x)}{a+b\cos^2(x)} \, dx$

Optimal. Leaf size=78

$$\frac{\left(a^2+3ab+3b^2\right)\cos(x)}{b^3}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}$$

```
[Out] (a^2+3*a*b+3*b^2)*cos(x)/b^3-1/3*(a+3*b)*cos(x)^3/b^2+1/5*cos(x)^5/b-(a+b)^
3*arctan(cos(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 390, 205}

$$\frac{\left(a^2+3ab+3b^2\right)\cos(x)}{b^3}-\frac{(a+3b)\cos^3(x)}{3b^2}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{\cos^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)^3*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2))) + ((a^2 +
 3*a*b + 3*b^2)*Cos[x])/b^3 - ((a + 3*b)*Cos[x]^3)/(3*b^2) + Cos[x]^5/(5*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a+bx^2}\,dx,x,\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{a^2+3ab+3b^2}{b^3}+\frac{(a+3b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3+3a^2b+3ab^2+b^3}{b^3\left(a+bx^2\right)}\right)dx,x,\cos(x)\right)\\
&= \frac{\left(a^2+3ab+3b^2\right)\cos(x)}{b^3}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}-\frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cos(\right.}{b^3}\\
&= -\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{\left(a^2+3ab+3b^2\right)\cos(x)}{b^3}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 143, normalized size = 1.83

$$\frac{(8a^2+22ab+19b^2)\cos(x)}{8b^3}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(4a+9b)\cos(}{48b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)^3*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/(Sqrt[a]*b^(
7/2))) - ((a + b)^3*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/(Sqrt
[a]*b^(7/2)) + ((8*a^2 + 22*a*b + 19*b^2)*Cos[x])/(8*b^3) - ((4*a + 9*b)*Co
s[3*x])/(48*b^2) + Cos[5*x]/(80*b)
```

fricas [A] time = 0.71, size = 225, normalized size = 2.88

$$\left[\frac{6\,ab^3\cos(x)^5-10\left(a^2b^2+3\,ab^3\right)\cos(x)^3-15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\cos(x)-a}{b\cos(x)^2+a}\right)+}{30\,ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*cos(x)^5 - 10*(a^2*b^2 + 3*a*b^3)*cos(x)^3 - 15*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos(x) - a)/
(b*cos(x)^2 + a)) + 30*(a^3*b + 3*a^2*b^2 + 3*a*b^3)*cos(x))/(a*b^4), 1/15*
(3*a*b^3*cos(x)^5 - 5*(a^2*b^2 + 3*a*b^3)*cos(x)^3 - 15*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a*b)*cos(x)/a) + 15*(a^3*b + 3*a^2*b^2 +
 3*a*b^3)*cos(x))/(a*b^4)]
```

giac [A] time = 0.15, size = 99, normalized size = 1.27

$$-\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^4\cos(x)^5-5\,ab^3\cos(x)^3-15\,b^4\cos(x)^3+15\,a^2b^2\cos(x)+45}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*b^4*cos(x)^5 - 5*a*b^3*cos(x)^3 - 15*b^4*cos(x)^3 + 15*a^2*b^2*c
os(x) + 45*a*b^3*cos(x) + 45*b^4*cos(x))/b^5
```

maple [B] time = 0.08, size = 138, normalized size = 1.77

$$\frac{\cos^5(x)}{5b}-\frac{(\cos^3(x))\,a}{3b^2}-\frac{\cos^3(x)}{b}+\frac{a^2\cos(x)}{b^3}+\frac{3a\cos(x)}{b^2}+\frac{3\cos(x)}{b}-\frac{\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)a^3}{b^3\sqrt{ab}}-\frac{3\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)a^2}{b^2\sqrt{ab}}-\frac{3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/(a+b*cos(x)^2),x)
```

```
[Out] 1/5*cos(x)^5/b-1/3/b^2*cos(x)^3*a-cos(x)^3/b+1/b^3*a^2*cos(x)+3*a*cos(x)/b^
2+3*cos(x)/b-1/b^3/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))*a^3-3/b^2/(a*b)
^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))*a^2-3/b/(a*b)^(1/2)*arctan(cos(x)*b/(a*
b)^(1/2))*a-1/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))
```

maxima [A]   time = 0.88, size = 87, normalized size = 1.12

$$-\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\arctan\left(\frac{b\cos\left(x\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{2}\cos\left(x\right)^{5}-5\,\left(ab+3\,b^{2}\right)\cos\left(x\right)^{3}+15\,\left(a^{2}+3\,ab+3\,b^{2}\right)\cos\left(x\right)}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*b^2*cos(x)^5 - 5*(a*b + 3*b^2)*cos(x)^3 + 15*(a^2 + 3*a*b + 3*b^
2)*cos(x))/b^3
```

mupad [B]   time = 2.13, size = 100, normalized size = 1.28

$$\cos(x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}+\frac{3}{b}\right)}{b}\right)-\cos(x)^3\left(\frac{a}{3\,b^2}+\frac{1}{b}\right)+\frac{\cos(x)^5}{5\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\cos(x)\,(a+b)^3}{\sqrt{a}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}\right)\,(a+b)^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/(a + b*cos(x)^2),x)
```

```
[Out] cos(x)*(3/b + (a*(a/b^2 + 3/b))/b) - cos(x)^3*(a/(3*b^2) + 1/b) + cos(x)^5/
(5*b) - (atan((b^(1/2)*cos(x)*(a + b)^3)/(a^(1/2)*(3*a*b^2 + 3*a^2*b + a^3
+ b^3)))*(a + b)^3)/(a^(1/2)*b^(7/2))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**7/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.11 $\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}$$

```
[Out] (a+2*b)*cos(x)/b^2-1/3*cos(x)^3/b-(a+b)^2*arctan(cos(x)*b^(1/2)/a^(1/2))/b^
(5/2)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 390, 205}

$$\frac{(a+2b)\cos(x)}{b^2}-\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{\cos^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)^2*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2))) + ((a + 2
*b)*Cos[x])/b^2 - Cos[x]^3/(3*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a+bx^2}\,dx,x,\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2\left(a+bx^2\right)}\right)dx,x,\cos(x)\right)\\
&= \frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}-\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cos(x)\right)}{b^2}\\
&= -\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}
\end{aligned}$$

Mathematica [B] time = 0.18, size = 116, normalized size = 2.15

$$\frac{3\sqrt{b}\,(4a+7b)\cos(x)-\frac{12(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{12(a+b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}}-b^{3/2}\cos(3x)}{12b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] ((-12*(a + b)^2*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/Sqrt[a] -
 (12*(a + b)^2*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/Sqrt[a] +
3*Sqrt[b]*(4*a + 7*b)*Cos[x] - b^(3/2)*Cos[3*x])/(12*b^(5/2))
```

fricas [A] time = 1.30, size = 152, normalized size = 2.81

$$\left[-\frac{2\,ab^2\cos(x)^3+3\left(a^2+2\,ab+b^2\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\,\cos(x)-a}{b\cos(x)^2+a}\right)-6\left(a^2b+2\,ab^2\right)\cos(x)}{6\,ab^3},-\frac{ab^2\cos(x)^3+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a*b^2*cos(x)^3 + 3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2
 + 2*sqrt(-a*b)*cos(x) - a)/(b*cos(x)^2 + a)) - 6*(a^2*b + 2*a*b^2)*cos(x))
/(a*b^3), -1/3*(a*b^2*cos(x)^3 + 3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(sqr
t(a*b)*cos(x)/a) - 3*(a^2*b + 2*a*b^2)*cos(x))/(a*b^3)]
```

giac [A] time = 0.17, size = 59, normalized size = 1.09

$$-\frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{b^2\cos(x)^3-3\,ab\cos(x)-6\,b^2\cos(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(a^2 + 2*a*b + b^2)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/3*(b^2*
cos(x)^3 - 3*a*b*cos(x) - 6*b^2*cos(x))/b^3
```

maple [A] time = 0.07, size = 86, normalized size = 1.59

$$-\frac{\cos^3(x)}{3b}+\frac{a\cos(x)}{b^2}+\frac{2\cos(x)}{b}-\frac{\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)a^2}{b^2\sqrt{ab}}-\frac{2\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)a}{b\sqrt{ab}}-\frac{\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+b*cos(x)^2),x)
```

```
[Out] -1/3*cos(x)^3/b+a*cos(x)/b^2+2*cos(x)/b-1/b^2/(a*b)^(1/2)*arctan(cos(x)*b/(
a*b)^(1/2))*a^2-2/b/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))*a-1/(a*b)^(1/2
)*arctan(cos(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.83, size = 53, normalized size = 0.98

$$-\frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{b\cos(x)^3-3\,(a+2\,b)\cos(x)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -(a^2 + 2*a*b + b^2)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/3*(b*co
s(x)^3 - 3*(a + 2*b)*cos(x))/b^2
```

mupad [B] time = 0.10, size = 65, normalized size = 1.20

$$\cos(x)\left(\frac{a}{b^2}+\frac{2}{b}\right)-\frac{\cos(x)^3}{3\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\cos(x)\,(a+b)^2}{\sqrt{a}\,\left(a^2+2\,a\,b+b^2\right)}\right)\,(a+b)^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a + b*cos(x)^2),x)
```

```
[Out] cos(x)*(a/b^2 + 2/b) - cos(x)^3/(3*b) - (atan((b^(1/2)*cos(x)*(a + b)^2)/(a
^(1/2)*(2*a*b + a^2 + b^2)))*(a + b)^2)/(a^(1/2)*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.12 $\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=36

$$\frac{\cos(x)}{b}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] cos(x)/b-(a+b)*arctan(cos(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 388, 205}

$$\frac{\cos(x)}{b}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + Cos[x]/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx,x,\cos(x)\right)\\ &= \frac{\cos(x)}{b}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cos(x)\right)}{b}\\ &= -\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{\cos(x)}{b}\end{aligned}$$

Mathematica [B] time = 0.18, size = 90, normalized size = 2.50

$$\frac{\sqrt{a}\sqrt{b}\cos(x)-\left((a+b)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)\right)-(a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] (-((a + b)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]]) - (a + b)*ArcT
an[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*Sqrt[b]*Cos[x])/(Sqr
t[a]*b^(3/2))
```

fricas [A] time = 0.55, size = 95, normalized size = 2.64

$$\left[\frac{2\,ab\cos(x) - \sqrt{-ab}\,(a+b)\log\left(-\frac{b\cos(x)^2 + 2\sqrt{-ab}\cos(x) - a}{b\cos(x)^2 + a}\right)}{2\,ab^2}, \frac{ab\cos(x) - \sqrt{ab}\,(a+b)\arctan\left(\frac{\sqrt{ab}\cos(x)}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*cos(x) - sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos
(x) - a)/(b*cos(x)^2 + a)))/(a*b^2), (a*b*cos(x) - sqrt(a*b)*(a + b)*arctan
(sqrt(a*b)*cos(x)/a))/(a*b^2)]
```

giac [A] time = 0.18, size = 30, normalized size = 0.83

$$-\frac{(a+b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{\cos(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(a + b)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b) + cos(x)/b
```

maple [A] time = 0.06, size = 46, normalized size = 1.28

$$\frac{\cos(x)}{b} - \frac{\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)a}{b\sqrt{ab}} - \frac{\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*cos(x)^2),x)
```

```
[Out] cos(x)/b-1/b/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))*a-1/(a*b)^(1/2)*arcta
n(cos(x)*b/(a*b)^(1/2))
```

maxima [A] time = 2.34, size = 30, normalized size = 0.83

$$-\frac{(a+b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{\cos(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -(a + b)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b) + cos(x)/b
```

mupad [B] time = 0.09, size = 28, normalized size = 0.78

$$\frac{\cos(x)}{b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)(a+b)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*cos(x)^2),x)
```

```
[Out] cos(x)/b - (atan((b^(1/2)*cos(x))/a^(1/2))*(a + b))/(a^(1/2)*b^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.13 $\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] -arctan(cos(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Cos[x]^2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x]^2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]))
```

fricas [A] time = 0.74, size = 73, normalized size = 2.81

$$\left[-\frac{\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\cos(x)-a}{b\cos(x)^2+a}\right)}{2ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\cos(x)}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos(x) - a)/(b*cos(x)^2 +
a))/(a*b), -sqrt(a*b)*arctan(sqrt(a*b)*cos(x)/a)/(a*b)]
```

giac [A] time = 0.18, size = 17, normalized size = 0.65

$$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(b*cos(x)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.04, size = 18, normalized size = 0.69

$$-\frac{\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)^2),x)
```

```
[Out] -1/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.92, size = 17, normalized size = 0.65

$$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -arctan(b*cos(x)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 2.23, size = 18, normalized size = 0.69

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x)^2),x)
```

```
[Out] -atan((b^(1/2)*cos(x))/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [A] time = 1.07, size = 87, normalized size = 3.35

$$\begin{cases} \frac{\tilde{\infty}}{\cos(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b\cos(x)} & \text{for } a = 0 \\ -\frac{\cos(x)}{a} & \text{for } b = 0 \\ \frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\cos(x)\right)}{2\sqrt{a}\,b\sqrt{\frac{1}{b}}} - \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\cos(x)\right)}{2\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)**2),x)
```

```
[Out] Piecewise((zoo/cos(x), Eq(a, 0) & Eq(b, 0)), (1/(b*cos(x)), Eq(a, 0)), (-co
s(x)/a, Eq(b, 0)), (I*log(-I*sqrt(a)*sqrt(1/b) + cos(x))/(2*sqrt(a)*b*sqrt(
1/b)) - I*log(I*sqrt(a)*sqrt(1/b) + cos(x))/(2*sqrt(a)*b*sqrt(1/b)), True))
```

3.14 $\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}-\frac{\tanh^{-1}(\cos(x))}{a+b}$$

```
[Out] -arctanh(cos(x))/(a+b)-arctan(cos(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3190, 391, 206, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}-\frac{\tanh^{-1}(\cos(x))}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Cos[x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b))) - ArcTanh[C
os[x]]/(a + b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)}\,dx, x, \cos(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)}{a+b} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)}{a+b}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} - \frac{\tanh^{-1}(\cos(x))}{a+b}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 1.19

$$\frac{-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}} + \log(1-\cos(x)) - \log(\cos(x)+1)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ((-2*Sqrt[b]*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/Sqrt[a] + Log[1 - Cos[x]] -
Log[1 + Cos[x]])/(2*(a + b))
```

fricas [A] time = 0.75, size = 113, normalized size = 2.69

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right) - \log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a+b)}, -\frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\cos(x)\right) \cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x) - a)/(b*cos(x)^2 +
 a)) - log(1/2*cos(x) + 1/2) + log(-1/2*cos(x) + 1/2))/(a + b), -1/2*(2*sqr
t(b/a)*arctan(sqrt(b/a)*cos(x)) + log(1/2*cos(x) + 1/2) - log(-1/2*cos(x) +
 1/2))/(a + b)]
```

giac [A] time = 0.22, size = 50, normalized size = 1.19

$$-\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log(\cos(x)+1)}{2(a+b)} + \frac{\log(-\cos(x)+1)}{2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*log(cos(x) + 1)/(a
+ b) + 1/2*log(-cos(x) + 1)/(a + b)
```

maple [A] time = 0.10, size = 56, normalized size = 1.33

$$-\frac{b\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}} + \frac{\ln(-1+\cos(x))}{2a+2b} - \frac{\ln(\cos(x)+1)}{2a+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*cos(x)^2),x)
```

```
[Out] -b/(a+b)/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))+1/(2*a+2*b)*ln(-1+cos(x))
-1/(2*a+2*b)*ln(cos(x)+1)
```

maxima [A] time = 1.72, size = 48, normalized size = 1.14

$$-\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}-\frac{\log\left(\cos(x)+1\right)}{2\,(a+b)}+\frac{\log\left(\cos(x)-1\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*log(cos(x) + 1)/(a
+ b) + 1/2*log(cos(x) - 1)/(a + b)
```

mupad [B] time = 2.67, size = 853, normalized size = 20.31

$$\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\cos(x)\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}+4\,b^3\,\cos(x)\right)1\mathrm{i}}{2\,(a+b)}-\frac{\left(\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\cos(x)\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}-4\,b^3\,\cos(x)\right)1\mathrm{i}}{2\,(a+b)}}{\frac{\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\cos(x)\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}+4\,b^3\,\cos(x)}{2\,(a+b)}+\frac{\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\cos(x)\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}-4\,b^3\,\cos(x)}{2\,(a+b)}}\right)1\mathrm{i}}{a+b}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*cos(x)^2)),x)
```

```
[Out] (atan(((((8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^
3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)) + 4*b^3*cos(x))*1i)/(2*(a + b)) -
(((8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a
^3*b^2))/(2*(a + b)))/(2*(a + b)) - 4*b^3*cos(x))*1i)/(2*(a + b)))/(((8*a*b
^3 + 4*b^4 + 4*a^2*b^2 - (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))
/(2*(a + b)))/(2*(a + b)) + 4*b^3*cos(x))/(2*(a + b)) + ((8*a*b^3 + 4*b^4 +
 4*a^2*b^2 + (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b))
)/(2*(a + b)) - 4*b^3*cos(x))/(2*(a + b))))*1i)/(a + b) + (atan((((-a*b)^(1
/2)*(2*b^3*cos(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (cos(x)*(-
a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*
(a*b + a^2)))*1i)/(a*b + a^2) + ((-a*b)^(1/2)*(2*b^3*cos(x) - ((-a*b)^(1/2)
*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a
^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*1i)/(a*b + a^2))/((
(-a*b)^(1/2)*(2*b^3*cos(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (
cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^
2))))/(2*(a*b + a^2))))/(a*b + a^2) - ((-a*b)^(1/2)*(2*b^3*cos(x) - ((-a*b)
^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5
 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2))))/(a*b + a^2))
)*(-a*b)^(1/2)*1i)/(a*(a + b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)**2),x)
[Out] Integral(csc(x)/(a + b*cos(x)**2), x)
```

3.15 $\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=62

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}-\frac{(a+3b)\tanh^{-1}(\cos(x))}{2(a+b)^2}-\frac{\cot(x)\csc(x)}{2(a+b)}$$

```
[Out] -1/2*(a+3*b)*arctanh(cos(x))/(a+b)^2-1/2*cot(x)*csc(x)/(a+b)-b^(3/2)*arctan
(cos(x)*b^(1/2)/a^(1/2))/(a+b)^2/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3190, 414, 522, 206, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}-\frac{(a+3b)\tanh^{-1}(\cos(x))}{2(a+b)^2}-\frac{\cot(x)\csc(x)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2)) - ((a + 3
*b)*ArcTanh[Cos[x]])/(2*(a + b)^2) - (Cot[x]*Csc[x])/(2*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx,x,\cos(x)\right)$$

$$= -\frac{\cot(x)\csc(x)}{2(a+b)} - \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx,x,\cos(x)\right)}{2(a+b)}$$

$$= -\frac{\cot(x)\csc(x)}{2(a+b)} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cos(x)\right)}{(a+b)^2} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(x)\right)}{2(a+b)^2}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2} - \frac{(a+3b)\tanh^{-1}(\cos(x))}{2(a+b)^2} - \frac{\cot(x)\csc(x)}{2(a+b)}$$

Mathematica [B] time = 0.55, size = 140, normalized size = 2.26

$$\frac{-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right) - 8b^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a}}\right) + \sqrt{a}\left(-\left((a+b)\csc^2\left(\frac{x}{2}\right)\right) + (a+b)\sec^2\left(\frac{x}{2}\right) - 4\right.}{8\sqrt{a}\,(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] (-8*b^(3/2)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]] - 8*b^(3/2)*Ar
cTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*(-((a + b)*Csc[x/2
]^2) - 4*(a + 3*b)*(Log[Cos[x/2]] - Log[Sin[x/2]]) + (a + b)*Sec[x/2]^2))/(
8*Sqrt[a]*(a + b)^2)
```

fricas [B] time = 0.67, size = 274, normalized size = 4.42

$$\left[\frac{2\left(b\cos(x)^2-b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right) + 2(a+b)\cos(x) - \left((a+3b)\cos(x)^2-a-3b\right)\log\left(\frac{1}{2}\cos\right.}{4\left(\left(a^2+2ab+b^2\right)\cos(x)^2-a^2-2ab-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*cos(x)^2 - b)*sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x)
 - a)/(b*cos(x)^2 + a)) + 2*(a + b)*cos(x) - ((a + 3*b)*cos(x)^2 - a - 3*b)
*log(1/2*cos(x) + 1/2) + ((a + 3*b)*cos(x)^2 - a - 3*b)*log(-1/2*cos(x) + 1
/2))/((a^2 + 2*a*b + b^2)*cos(x)^2 - a^2 - 2*a*b - b^2), -1/4*(4*(b*cos(x)^
2 - b)*sqrt(b/a)*arctan(sqrt(b/a)*cos(x)) - 2*(a + b)*cos(x) + ((a + 3*b)*c
os(x)^2 - a - 3*b)*log(1/2*cos(x) + 1/2) - ((a + 3*b)*cos(x)^2 - a - 3*b)*l
og(-1/2*cos(x) + 1/2))/((a^2 + 2*a*b + b^2)*cos(x)^2 - a^2 - 2*a*b - b^2)]
```

giac [B] time = 0.17, size = 103, normalized size = 1.66

$$-\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\left(a^2+2ab+b^2\right)\sqrt{ab}} - \frac{(a+3b)\log(\cos(x)+1)}{4\left(a^2+2ab+b^2\right)} + \frac{(a+3b)\log(-\cos(x)+1)}{4\left(a^2+2ab+b^2\right)} + \frac{\cos(x)}{2\left(\cos(x)^2-1\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(b*cos(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 1/4*(a +
3*b)*log(cos(x) + 1)/(a^2 + 2*a*b + b^2) + 1/4*(a + 3*b)*log(-cos(x) + 1)/(
a^2 + 2*a*b + b^2) + 1/2*cos(x)/((cos(x)^2 - 1)*(a + b))
```

maple [B] time = 0.12, size = 111, normalized size = 1.79

$$-\frac{b^2 \arctan\left(\frac{\cos(x) b}{\sqrt{ab}}\right)}{(a+b)^2 \sqrt{ab}}+\frac{1}{(4a+4b)(-1+\cos(x))}+\frac{\ln(-1+\cos(x))\, a}{4(a+b)^2}+\frac{3\ln(-1+\cos(x))\, b}{4(a+b)^2}+\frac{1}{(4a+4b)(\cos(x)+1)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*cos(x)^2),x)
```

```
[Out] -b^2/(a+b)^2/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))+1/(4*a+4*b)/(-1+cos(x
))+1/4/(a+b)^2*ln(-1+cos(x))*a+3/4/(a+b)^2*ln(-1+cos(x))*b+1/(4*a+4*b)/(cos
(x)+1)-1/4/(a+b)^2*ln(cos(x)+1)*a-3/4/(a+b)^2*ln(cos(x)+1)*b
```

maxima [B] time = 1.18, size = 105, normalized size = 1.69

$$-\frac{b^2 \arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{ab}}-\frac{(a+3\,b)\log(\cos(x)+1)}{4\left(a^2+2\,ab+b^2\right)}+\frac{(a+3\,b)\log(\cos(x)-1)}{4\left(a^2+2\,ab+b^2\right)}+\frac{\cos(x)}{2\left((a+b)\cos(x)^2-a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b^2*arctan(b*cos(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 1/4*(a +
3*b)*log(cos(x) + 1)/(a^2 + 2*a*b + b^2) + 1/4*(a + 3*b)*log(cos(x) - 1)/(a
^2 + 2*a*b + b^2) + 1/2*cos(x)/((a + b)*cos(x)^2 - a - b)
```

mupad [B] time = 2.59, size = 1138, normalized size = 18.35

$$\ln(\cos(x)-1)\left(\frac{b}{2(a+b)^2}+\frac{1}{4(a+b)}\right)-\frac{\cos(x)}{2\sin(x)^2\,(a+b)}-\frac{\ln(\cos(x)+1)\,(a+3\,b)}{4(a+b)^2}-\frac{\operatorname{atan}\left(\frac{\sqrt{-a\,b^3}\left(\frac{\cos(x)\left(a^2\,b^3+\cdots\right)}{4\left(a^2+2\cdots\right)}\cdots\right)}{\cdots}\right)}{\frac{\frac{3\,b^5}{2}+\frac{a\,b^4}{2}}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*cos(x)^2)),x)
```

```
[Out] log(cos(x) - 1)*(b/(2*(a + b)^2) + 1/(4*(a + b))) - cos(x)/(2*sin(x)^2*(a +
 b)) - (log(cos(x) + 1)*(a + 3*b))/(4*(a + b)^2) - (atan((((-a*b^3)^(1/2)*(
(cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6
 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 +
3*a^2*b + a^3 + b^3)) - (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*
b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2
+ 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2
+ 2*a^2*b + a^3) + ((-a*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(
4*(2*a*b + a^2 + b^2)) - (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12
*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (cos(x)*(-a*b^3
```

```
)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*
b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(
a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2 + 2*a^2*b + a^3))/(((a*b^4)/2 + (3*b^5)
/2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - ((-a*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 1
3*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6 + 4*b^7 + 32*a^2*b^
5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
) - (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 4
8*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(
-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))))/(a*b^2 + 2*a^2*b + a^3) + ((-a
*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2))
- (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(
2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*
b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 +
 b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))
))/(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2)*1i)/(a*b^2 + 2*a^2*b + a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**3/(a + b*cos(x)**2), x)
```

3.16 $\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\cos(x))}{8(a+b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}-\frac{\cot(x)\csc^3(x)}{4(a+b)}-\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2}$$

```
[Out] -1/8*(3*a^2+10*a*b+15*b^2)*arctanh(cos(x))/(a+b)^3-1/8*(3*a+7*b)*cot(x)*csc
(x)/(a+b)^2-1/4*cot(x)*csc(x)^3/(a+b)-b^(5/2)*arctan(cos(x)*b^(1/2)/a^(1/2)
)/(a+b)^3/a^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3190, 414, 527, 522, 206, 205}

$$-\frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\cos(x))}{8(a+b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}-\frac{\cot(x)\csc^3(x)}{4(a+b)}-\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3)) - ((3*a^2
 + 10*a*b + 15*b^2)*ArcTanh[Cos[x]])/(8*(a + b)^3) - ((3*a + 7*b)*Cot[x]*Cs
c[x])/(8*(a + b)^2) - (Cot[x]*Csc[x]^3)/(4*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+bx^2\right)}\,dx, x, \cos(x)\right)$$

$$= -\frac{\cot(x)\csc^3(x)}{4(a+b)} - \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \cos(x)\right)}{4(a+b)}$$

$$= -\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2} - \frac{\cot(x)\csc^3(x)}{4(a+b)} - \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \cos(x)\right)}{8(a+b)^2}$$

$$= -\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2} - \frac{\cot(x)\csc^3(x)}{4(a+b)} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)}{(a+b)^3} - \frac{\left(3a^2+1 \right.}{}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3} - \frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\cos(x))}{8(a+b)^3} - \frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2}$$

Mathematica [B] time = 1.47, size = 204, normalized size = 2.17

$$\frac{\sqrt{a}\left(-2\left(3a^2+10ab+7b^2\right)\csc^2\left(\frac{x}{2}\right)+2\left(3a^2+10ab+7b^2\right)\sec^2\left(\frac{x}{2}\right)-8\left(3a^2+10ab+15b^2\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\right.\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] (-64*b^(5/2)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]] - 64*b^(5/2)*
ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*(-2*(3*a^2 + 10*
a*b + 7*b^2)*Csc[x/2]^2 - (a + b)^2*Csc[x/2]^4 - 8*(3*a^2 + 10*a*b + 15*b^2
)*(Log[Cos[x/2]] - Log[Sin[x/2]]) + 2*(3*a^2 + 10*a*b + 7*b^2)*Sec[x/2]^2 +
 (a + b)^2*Sec[x/2]^4))/(64*Sqrt[a]*(a + b)^3)
```

fricas [B] time = 0.87, size = 592, normalized size = 6.30

$$\left[\frac{2\left(3a^2+10ab+7b^2\right)\cos(x)^3+8\left(b^2\cos(x)^4-2b^2\cos(x)^2+b^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right)-2\left(5a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 + 10*a*b + 7*b^2)*cos(x)^3 + 8*(b^2*cos(x)^4 - 2*b^2*cos(x)
^2 + b^2)*sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x) - a)/(b*cos(x)
^2 + a)) - 2*(5*a^2 + 14*a*b + 9*b^2)*cos(x) - ((3*a^2 + 10*a*b + 15*b^2)*c
os(x)^4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*l
og(1/2*cos(x) + 1/2) + ((3*a^2 + 10*a*b + 15*b^2)*cos(x)^4 - 2*(3*a^2 + 10*
a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*log(-1/2*cos(x) + 1/2))/(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2), 1/16*(2*(3*a^2 + 10*a*b + 7*b^
2)*cos(x)^3 - 16*(b^2*cos(x)^4 - 2*b^2*cos(x)^2 + b^2)*sqrt(b/a)*arctan(sqr
t(b/a)*cos(x)) - 2*(5*a^2 + 14*a*b + 9*b^2)*cos(x) - ((3*a^2 + 10*a*b + 15*
b^2)*cos(x)^4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*
b^2)*log(1/2*cos(x) + 1/2) + ((3*a^2 + 10*a*b + 15*b^2)*cos(x)^4 - 2*(3*a^2
 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*log(-1/2*cos(x) + 1
/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2)]
```

giac [B] time = 0.19, size = 178, normalized size = 1.89

$$-\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab}}-\frac{\left(3a^2+10ab+15b^2\right)\log\left(\cos(x)+1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\left(3a^2+10ab+15b^2\right)\log\left(-\cos(x)+1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*cos(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
 - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-cos(x) + 1)/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 1/8*(3*a*cos(x)^3 + 7*b*cos(x)^3 - 5*a*cos(x) - 9*b*cos(x))
/((a^2 + 2*a*b + b^2)*(cos(x)^2 - 1)^2)
```

maple [B] time = 0.13, size = 205, normalized size = 2.18

$$-\frac{b^3\arctan\left(\frac{\cos(x)b}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}-\frac{1}{2\left(8a+8b\right)\left(-1+\cos(x)\right)^2}+\frac{3a}{16\left(a+b\right)^2\left(-1+\cos(x)\right)}+\frac{7b}{16\left(a+b\right)^2\left(-1+\cos(x)\right)}+\frac{3\ln(-1+}{16(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(a+b*cos(x)^2),x)
```

```
[Out] -b^3/(a+b)^3/(a*b)^(1/2)*arctan(cos(x)*b/(a*b)^(1/2))-1/2/(8*a+8*b)/(-1+cos
(x))^2+3/16/(a+b)^2/(-1+cos(x))*a+7/16/(a+b)^2/(-1+cos(x))*b+3/16/(a+b)^3*l
n(-1+cos(x))*a^2+5/8/(a+b)^3*ln(-1+cos(x))*a*b+15/16/(a+b)^3*ln(-1+cos(x))*
b^2+1/2/(8*a+8*b)/(cos(x)+1)^2+3/16/(a+b)^2/(cos(x)+1)*a+7/16/(a+b)^2/(cos(
x)+1)*b-3/16/(a+b)^3*ln(cos(x)+1)*a^2-5/8/(a+b)^3*ln(cos(x)+1)*a*b-15/16/(a
+b)^3*ln(cos(x)+1)*b^2
```

maxima [B] time = 0.73, size = 200, normalized size = 2.13

$$-\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab}}-\frac{\left(3a^2+10ab+15b^2\right)\log\left(\cos(x)+1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\left(3a^2+10ab+15b^2\right)\log\left(\cos(x)-1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*cos(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
 - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2
```

```
+ b^3) + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) - 1)/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3) + 1/8*((3*a + 7*b)*cos(x)^3 - (5*a + 9*b)*cos(x))/((a^2 + 2*a*
b + b^2)*cos(x)^4 - 2*(a^2 + 2*a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)
```

mupad [B] time = 5.28, size = 833, normalized size = 8.86

$$-\frac{3\,a^3\,\mathrm{atanh}\left(\cos(x)\right)-3\,a^3\,{\cos(x)}^3+5\,a^3\,\cos(x)+9\,a\,b^2\,\cos(x)+14\,a^2\,b\,\cos(x)-6\,a^3\,\mathrm{atanh}\left(\cos(x)\right)\,\cos(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5*(a + b*cos(x)^2)),x)
```

```
[Out] -(atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a*b^5)^(3/2)*64i + a^6*b*c
os(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*cos(
x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*cos(x
)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5
 + 60*a^6*b^4 + 9*a^7*b^3))*(-a*b^5)^(1/2)*8i - 3*a^3*cos(x)^3 + 3*a^3*atan
h(cos(x)) + 5*a^3*cos(x) - atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a
*b^5)^(3/2)*64i + a^6*b*cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^
(1/2)*289i + a^3*b^4*cos(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(
1/2)*190i + a^5*b^2*cos(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 +
300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*cos(x)^2*(-a*b^5)^(1/2
)*16i + atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a*b^5)^(3/2)*64i + a
^6*b*cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^(1/2)*289i + a^3*b^
4*cos(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(1/2)*190i + a^5*b^2
*cos(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a
^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*cos(x)^4*(-a*b^5)^(1/2)*8i + 9*a*b^2*cos(
x) + 14*a^2*b*cos(x) - 6*a^3*atanh(cos(x))*cos(x)^2 + 3*a^3*atanh(cos(x))*c
os(x)^4 - 7*a*b^2*cos(x)^3 - 10*a^2*b*cos(x)^3 + 15*a*b^2*atanh(cos(x)) + 1
0*a^2*b*atanh(cos(x)) - 30*a*b^2*atanh(cos(x))*cos(x)^2 - 20*a^2*b*atanh(co
s(x))*cos(x)^2 + 15*a*b^2*atanh(cos(x))*cos(x)^4 + 10*a^2*b*atanh(cos(x))*c
os(x)^4)/(8*a^4*cos(x)^4 - 16*a^4*cos(x)^2 + 8*a*b^3 + 24*a^3*b + 8*a^4 + 2
4*a^2*b^2 - 48*a^2*b^2*cos(x)^2 + 24*a^2*b^2*cos(x)^4 - 16*a*b^3*cos(x)^2 -
 48*a^3*b*cos(x)^2 + 8*a*b^3*cos(x)^4 + 24*a^3*b*cos(x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(x)}{a + b\cos^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**5/(a + b*cos(x)**2), x)
```

3.17 $\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=88

$$-\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}+\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}+\frac{\sin^3(x)\cos(x)}{4b}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*x/b^3+1/8*(4*a+7*b)*cos(x)*sin(x)/b^2+1/4*cos(x)
*sin(x)^3/b-(a+b)^(5/2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/b^3/a^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3191, 414, 527, 522, 203, 205}

$$-\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}+\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}+\frac{\sin^3(x)\cos(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] -((8*a^2 + 20*a*b + 15*b^2)*x)/(8*b^3) - ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]
*Cot[x])/Sqrt[a]])/(Sqrt[a]*b^3) + ((4*a + 7*b)*Cos[x]*Sin[x])/(8*b^2) + (C
os[x]*Sin[x]^3)/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)$$

$$= \frac{\cos(x)\sin^3(x)}{4b} - \frac{\text{Subst}\left(\int \frac{a+4b-3(a+b)x^2}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)}{4b}$$

$$= \frac{(4a+7b)\cos(x)\sin(x)}{8b^2} + \frac{\cos(x)\sin^3(x)}{4b} - \frac{\text{Subst}\left(\int \frac{4a^2+9ab+8b^2-(a+b)(4a+7b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \cot(\right.}{8b^2}$$

$$= \frac{(4a+7b)\cos(x)\sin(x)}{8b^2} + \frac{\cos(x)\sin^3(x)}{4b} - \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b^3} + -$$

$$= -\frac{\left(8a^2+20ab+15b^2\right)x}{8b^3} - \frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3} + \frac{(4a+7b)\cos(x)\sin(x)}{8b^2} + \cos$$

Mathematica [A] time = 0.20, size = 77, normalized size = 0.88

$$\frac{-4x\left(8a^2+20ab+15b^2\right)+8b(a+2b)\sin(2x)+\frac{32(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}-b^2\sin(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (-4*(8*a^2 + 20*a*b + 15*b^2)*x + (32*(a + b)^(5/2)*ArcTan[(Sqrt[a]*Tan[x])
/Sqrt[a + b]])/Sqrt[a] + 8*b*(a + 2*b)*Sin[2*x] - b^2*Sin[4*x])/(32*b^3)
```

fricas [A] time = 2.47, size = 285, normalized size = 3.24

$$\left[\frac{2\left(a^2+2ab+b^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2-4\left(\left(2a^2+ab\right)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-(8}{8b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*cos(x)
```

```
)*sqrt(-(a + b)/a)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - (
8*a^2 + 20*a*b + 15*b^2)*x - (2*b^2*cos(x)^3 - (4*a*b + 9*b^2)*cos(x))*sin(
x))/b^3, -1/8*(4*(a^2 + 2*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*
cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*a*b +
15*b^2)*x + (2*b^2*cos(x)^3 - (4*a*b + 9*b^2)*cos(x))*sin(x))/b^3]
```

giac [A] time = 1.61, size = 119, normalized size = 1.35

$$-\frac{\left(8\,a^2+20\,ab+15\,b^2\right)x}{8\,b^3}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,b^3}+\frac{4\,a\tan(x)^3+9\,b\tan(}{8\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*
floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*
b)*b^3) + 1/8*(4*a*tan(x)^3 + 9*b*tan(x)^3 + 4*a*tan(x) + 7*b*tan(x))/((tan
(x)^2 + 1)^2*b^2)
```

maple [B] time = 0.09, size = 194, normalized size = 2.20

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a^3}{b^3\sqrt{(a+b)\,a}}+\frac{3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a^2}{b^2\sqrt{(a+b)\,a}}+\frac{3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a}{b\sqrt{(a+b)\,a}}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)\,a}}+\frac{\left(\tan^3(x)\right)a}{2b^2\left(\tan^2(x)+1\right)^2}+\frac{9\left(\mathrm{t}\right.}{8b\left(\mathrm{tan}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a+b*cos(x)^2),x)
```

```
[Out] 1/b^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a^3+3/b^2/((a+b)*a)^
(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a^2+3/b/((a+b)*a)^(1/2)*arctan(a*tan
(x)/((a+b)*a)^(1/2))*a+1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))+1
/2/b^2/(tan(x)^2+1)^2*tan(x)^3*a+9/8/b/(tan(x)^2+1)^2*tan(x)^3+1/2/b^2/(tan
(x)^2+1)^2*tan(x)*a+7/8/b/(tan(x)^2+1)^2*tan(x)-1/b^3*arctan(tan(x))*a^2-5/
2/b^2*arctan(tan(x))*a-15/8/b*arctan(tan(x))
```

maxima [A] time = 1.03, size = 112, normalized size = 1.27

$$\frac{(4\,a+9\,b)\tan(x)^3+(4\,a+7\,b)\tan(x)}{8\left(b^2\tan(x)^4+2\,b^2\tan(x)^2+b^2\right)}-\frac{\left(8\,a^2+20\,ab+15\,b^2\right)x}{8\,b^3}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((4*a + 9*b)*tan(x)^3 + (4*a + 7*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(x
)^2 + b^2) - 1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3)
```

mupad [B] time = 2.68, size = 681, normalized size = 7.74

$$\frac{\frac{\tan(x)^3\,(4\,a+9\,b)}{8\,b^2}+\frac{\tan(x)\,(4\,a+7\,b)}{8\,b^2}}{\tan(x)^4+2\tan(x)^2+1}-\frac{\mathrm{atanh}\left(\frac{95\,a^2\tan(x)\,\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3\,b^3-5\,a^2\,b^4-a\,b^5}}{32\left(2\,a\,b^4+\frac{469\,a^4\,b}{32}+\frac{215\,a^5}{32}+\frac{287\,a^2\,b^3}{32}+\frac{517\,a^3\,b^2}{32}+\frac{5\,a^6}{4\,b}\right)}+\frac{5\,a^3\tan(x)\,\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3}}{4\left(\frac{5\,a^6}{4}+\frac{215\,a^5\,b}{32}+\frac{469\,a^4\,b^2}{32}+\frac{517\,a^3\,b^3}{32}+\right.}\right.}{a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a + b*cos(x)^2),x)
```

```
[Out] ((tan(x)^3*(4*a + 9*b))/(8*b^2) + (tan(x)*(4*a + 7*b))/(8*b^2))/(2*tan(x)^2
 + tan(x)^4 + 1) + (atan((5717*a^3*tan(x))/(256*((15*a*b^2)/4 + (3665*a^2*b
)/256 + (5717*a^3)/256 + (1143*a^4)/(64*b) + (235*a^5)/(32*b^2) + (5*a^6)/(
4*b^3))) + (3665*a^2*tan(x))/(256*((15*a*b)/4 + (3665*a^2)/256 + (5717*a^3)
/(256*b) + (1143*a^4)/(64*b^2) + (235*a^5)/(32*b^3) + (5*a^6)/(4*b^4))) + (
1143*a^4*tan(x))/(64*((15*a*b^3)/4 + (5717*a^3*b)/256 + (1143*a^4)/64 + (36
65*a^2*b^2)/256 + (235*a^5)/(32*b) + (5*a^6)/(4*b^2))) + (235*a^5*tan(x))/(
32*((15*a*b^4)/4 + (1143*a^4*b)/64 + (235*a^5)/32 + (3665*a^2*b^3)/256 + (5
717*a^3*b^2)/256 + (5*a^6)/(4*b))) + (5*a^6*tan(x))/(4*((15*a*b^5)/4 + (235
*a^5*b)/32 + (5*a^6)/4 + (3665*a^2*b^4)/256 + (5717*a^3*b^3)/256 + (1143*a^
4*b^2)/64)) + (15*a*b*tan(x))/(4*((15*a*b)/4 + (3665*a^2)/256 + (5717*a^3)/
(256*b) + (1143*a^4)/(64*b^2) + (235*a^5)/(32*b^3) + (5*a^6)/(4*b^4))))*(a*
b*20i + a^2*8i + b^2*15i)*1i)/(8*b^3) - (atanh((95*a^2*tan(x)*(- a*b^5 - 5*
a^5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/(32*(2*a*b^4 + (4
69*a^4*b)/32 + (215*a^5)/32 + (287*a^2*b^3)/32 + (517*a^3*b^2)/32 + (5*a^6)
/(4*b))) + (5*a^3*tan(x)*(- a*b^5 - 5*a^5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3
- 10*a^4*b^2)^(1/2))/(4*(2*a*b^5 + (215*a^5*b)/32 + (5*a^6)/4 + (287*a^2*b^
4)/32 + (517*a^3*b^3)/32 + (469*a^4*b^2)/32)) + (2*a*tan(x)*(- a*b^5 - 5*a^
5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/(2*a*b^3 + (517*a^3
*b)/32 + (469*a^4)/32 + (287*a^2*b^2)/32 + (215*a^5)/(32*b) + (5*a^6)/(4*b^
2)))*(-a*(a + b)^5)^(1/2))/(a*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.18 $\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{x(2a+3b)}{2b^2}-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}+\frac{\sin(x)\cos(x)}{2b}$$

```
[Out] -1/2*(2*a+3*b)*x/b^2+1/2*cos(x)*sin(x)/b-(a+b)^(3/2)*arctan(cot(x)*(a+b)^(1
/2)/a^(1/2))/b^2/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3191, 414, 522, 203, 205}

$$-\frac{x(2a+3b)}{2b^2}-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}+\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -((2*a + 3*b)*x)/(2*b^2) - ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[
a]])/(Sqrt[a]*b^2) + (Cos[x]*Sin[x])/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1}{(1+x^2)^2\,(a+(a+b)x^2)}\,dx,x,\cot(x)\right)$$
$$= \frac{\cos(x)\sin(x)}{2b} - \frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx,x,\cot(x)\right)}{2b}$$
$$= \frac{\cos(x)\sin(x)}{2b} - \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{b^2} + \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\right.}{2b^2}$$
$$= -\frac{(2a+3b)x}{2b^2} - \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2} + \frac{\cos(x)\sin(x)}{2b}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.87

$$\frac{\frac{4(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}} - 4ax - 6bx + b\sin(2x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (-4*a*x - 6*b*x + (4*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sq
rt[a] + b*Sin[2*x])/(4*b^2)
```

fricas [A] time = 0.62, size = 211, normalized size = 3.52

$$\left[\frac{2\,b\cos(x)\sin(x) + (a+b)\sqrt{-\frac{a+b}{a}}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4 - 2\,(4\,a^2+3\,ab)\cos(x)^2 - 4\,((2\,a^2+ab)\cos(x)^3 - a^2\cos(x))\sqrt{-\frac{a+b}{a}}\,\sin(x)}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right.}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*cos(x)*sin(x) + (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^
2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*
cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2
)) - 2*(2*a + 3*b)*x)/b^2, 1/2*(b*cos(x)*sin(x) - (a + b)*sqrt((a + b)/a)*a
rctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x)))
 - (2*a + 3*b)*x)/b^2]
```

giac [A] time = 0.19, size = 80, normalized size = 1.33

$$-\frac{(2\,a+3\,b)x}{2\,b^2} + \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(a) + \arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{a^2+ab}\,b^2} + \frac{\tan(x)}{2\left(\tan(x)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqr
t(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*b^2) + 1/2*tan(x)/((tan
(x)^2 + 1)*b)
```

maple [B] time = 0.08, size = 105, normalized size = 1.75

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a^2}{b^2\sqrt{(a+b)\,a}}+\frac{2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a}{b\sqrt{(a+b)\,a}}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)\,a}}+\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}-\frac{3\arctan(\tan(x))}{2b}-\frac{\arctan(\tan(x))a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*cos(x)^2),x)
```

```
[Out] 1/b^2/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a^2+2/b/((a+b)*a)^(1
/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a+1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((
a+b)*a)^(1/2))+1/2/b*tan(x)/(tan(x)^2+1)-3/2/b*arctan(tan(x))-1/b^2*arctan(
tan(x))*a
```

maxima [A] time = 0.77, size = 62, normalized size = 1.03

$$-\frac{(2\,a+3\,b)x}{2\,b^2}+\frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}+\frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + 1/2*tan(x)/(b*tan(x)^2 + b) + (a^2 + 2*a*b + b^2)*
arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2)
```

mupad [B] time = 2.45, size = 126, normalized size = 2.10

$$\frac{\cos(x)\,\sin(x)}{2\,b}-\frac{a\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{b^2}-\frac{3\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{2\,b}-\frac{\mathrm{atanh}\left(\frac{\sin(x)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{\cos(x)\,a^2+2\,\cos(x)\,a\,b+\cos(x)\,b^2}\right)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*cos(x)^2),x)
```

```
[Out] (cos(x)*sin(x))/(2*b) - (a*atan(sin(x)/cos(x)))/b^2 - (3*atan(sin(x)/cos(x)
))/(2*b) - (atanh((sin(x)*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a^2
*cos(x) + b^2*cos(x) + 2*a*b*cos(x)))*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)
^(1/2))/(a*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

```
[Out] -x/b-arctan(cot(x)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 391, 203, 205}

$$-\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] -(x/b) - (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx,x,\cot(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\cot(x)\right)}{b}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{b}\\
&= -\frac{x}{b}-\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.92

$$\frac{\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}-x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] (-x + (Sqrt[a + b]*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a])/b
```

fricas [A] time = 0.75, size = 177, normalized size = 4.42

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2-4\left(\left(2a^2+ab\right)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-4x}{4b}, -\frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{((\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*cos(x))*sqrt(-(a + b)/a)*sin
(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*x)/b, -1/2*(sqrt((a +
 b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*
sin(x))) + 2*x)/b]
```

giac [A] time = 0.19, size = 50, normalized size = 1.25

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(a + b)/(s
qrt(a^2 + a*b)*b) - x/b
```

maple [A] time = 0.07, size = 54, normalized size = 1.35

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a}{b\sqrt{(a+b)\,a}}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)\,a}}-\frac{\arctan(\tan(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*cos(x)^2),x)
```

```
[Out] 1/b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a+1/((a+b)*a)^(1/2)*ar
ctan(a*tan(x)/((a+b)*a)^(1/2))-1/b*arctan(tan(x))
```

maxima [A] time = 1.74, size = 33, normalized size = 0.82

$$\frac{(a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] (a + b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - x/b
```

mupad [B] time = 2.37, size = 108, normalized size = 2.70

$$-\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\,\tan(x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\,\tan(x)}{2\,a^2\,b+2\,a\,b^2}\right)}{b}-\frac{\operatorname{atanh}\left(\frac{2\,a^2\,b\,\tan(x)\,\sqrt{-a^2-b\,a}}{2\,a^3\,b+2\,a^2\,b^2}\right)\sqrt{-a\,(a+b)}}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*cos(x)^2),x)
```

```
[Out] - atan((2*a*b^2*tan(x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(x))/(2*a*b^2 + 2
*a^2*b))/b - (atanh((2*a^2*b*tan(x)*(- a*b - a^2)^(1/2))/(2*a^3*b + 2*a^2*b
^2))*(-a*(a + b))^(1/2))/(a*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.20 $\int \frac{1}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

```
[Out] -arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3181, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-1),x]
```

```
[Out] -(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

fricas [B] time = 0.58, size = 163, normalized size = 5.43

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{4\left(a^2+ab\right)}, -\frac{\arctan\left(\frac{(2a+b)\,\text{co}}{2\sqrt{a^2+ab}\,\text{co}}\right.}{2\sqrt{a^2+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2*((
2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

giac [A] time = 0.16, size = 37, normalized size = 1.23

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2 + ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2 +
 a*b)
```

maple [A] time = 0.06, size = 21, normalized size = 0.70

$$\frac{\arctan\left(\frac{a \tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a + b)\, a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2),x)
```

```
[Out] 1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 0.95, size = 20, normalized size = 0.67

$$\frac{\arctan\left(\frac{a \tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a + b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

mupad [B] time = 2.38, size = 24, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{a \tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2 + b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2),x)
```

```
[Out] atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

sympy [A] time = 38.28, size = 12026, normalized size = 400.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-tan(x/2)
/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (-2*tan(x/2)/(b*(tan(x/2)**2 - 1)),
Eq(a, 0)), (-5*I*a**(5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
+ b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(
a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) +
 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
 b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
+ b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + 5*I*a**(
5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log
(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*
I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b
))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)
*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) -
 a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
+ b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 3*I*a**(5/2)*sqrt(b)*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*s
qrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) -
 a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b
))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2
*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))) + 3*I*a**(5/2)*sqrt(b)*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)))
+ 10*I*a**(3/2)*b**(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + t
an(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) +
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
```

```
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(
a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
 + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 10*I*a**(3/2)*b**
(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(sqrt(2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7
/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/
2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
 b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
 - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))) - 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
 + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
 - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b))) + 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - I*s
qrt(a)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*
log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/
(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(
3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sq
rt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b
/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b
**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + I*sqrt(a)*b**(5/2)*sqrt(-2
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*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(
a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**
2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a
/(a + b) + b/(a + b))) + I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)))
 - I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/
2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*
a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2
*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + a**3*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) -
 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
+ b/(a + b))) - a**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
 b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/
2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*
a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
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) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2
*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + a**3*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) -
 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
+ b/(a + b))) - a**3*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/
2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*
a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2
*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 10*a**2*b*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b))) + 10*a**2*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a
**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) -
 a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 2*a**2*b*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(
a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt
(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
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(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
+ b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**
2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(
a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) - a/(a + b) + b/(a + b))) + 2*a**2*b*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a
/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b
/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
 b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(
a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b
)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b
/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + 5*a*
b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7
/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/
2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
 b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
- 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))) - 5*a*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(
a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
) - 3*a*b**2*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(
-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8
*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2
)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
 b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(
a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3
*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + 3*a*b**2*sqrt(2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
```

```
 - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
 + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*
a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*s
qrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b))), True))
```

3.21 $\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=41

$$-\frac{\cot(x)}{a+b}-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}$$

```
[Out] -cot(x)/(a+b)-b*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/(a+b)^(3/2)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 388, 205}

$$-\frac{\cot(x)}{a+b}-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2))) - Cot[x
]/(a + b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx,x,\cot(x)\right)\\ &= -\frac{\cot(x)}{a+b}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{a+b}\\ &= -\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}-\frac{\cot(x)}{a+b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 40, normalized size = 0.98

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}}-\frac{\cot(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(3/2)) - Cot[x]/(
a + b)
```

fricas [B] time = 0.67, size = 228, normalized size = 5.56

$$\left[-\frac{\sqrt{-a^2-ab}\,b\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+3\,ab\right)\cos(x)^2+4\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)\sin(x)+4\left(a^2+\right.}{4\left(a^3+2\,a^2b+ab^2\right)\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 +
3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
 + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) + 4*(a^2 + a*b)*cos(x
))/((a^3 + 2*a^2*b + a*b^2)*sin(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2
*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*sin(x) + 2*(a^2 + a*
b)*cos(x))/((a^3 + 2*a^2*b + a*b^2)*sin(x))]
```

giac [A] time = 0.34, size = 55, normalized size = 1.34

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}\,(a+b)}-\frac{1}{(a+b)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b/(sqrt(a^
2 + a*b)*(a + b)) - 1/((a + b)*tan(x))
```

maple [A] time = 0.11, size = 39, normalized size = 0.95

$$\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a+b)\sqrt{(a+b)\,a}}-\frac{1}{(a+b)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cos(x)^2),x)
```

```
[Out] b/(a+b)/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))-1/(a+b)/tan(x)
```

maxima [A] time = 0.74, size = 38, normalized size = 0.93

$$\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)}-\frac{1}{(a+b)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)) - 1/((a + b)*t
an(x))
```

mupad [B] time = 2.30, size = 34, normalized size = 0.83

$$\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}}-\frac{1}{\tan(x)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*cos(x)^2)),x)
```

```
[Out] (b*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(1/2)*(a + b)^(3/2)) - 1/(tan(x
)*(a + b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cos(x)**2), x)
```

3.22 $\int \frac{\csc^4(x)}{a+b\cos^2(x)} dx$

Optimal. Leaf size=61

$$-\frac{b^2 \tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{\cot^3(x)}{3(a+b)}-\frac{(a+2b)\cot(x)}{(a+b)^2}$$

```
[Out] -(a+2*b)*cot(x)/(a+b)^2-1/3*cot(x)^3/(a+b)-b^2*arctan(cot(x)*(a+b)^(1/2)/a^
(1/2))/(a+b)^(5/2)/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 390, 205}

$$-\frac{b^2 \tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{\cot^3(x)}{3(a+b)}-\frac{(a+2b)\cot(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^2*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2))) - ((a
 + 2*b)*Cot[x])/(a + b)^2 - Cot[x]^3/(3*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+(a+b)x^2}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2\left(a+(a+b)x^2\right)}\right)dx, x, \cot(x)\right)$$
$$= -\frac{(a+2b)\cot(x)}{(a+b)^2}-\frac{\cot^3(x)}{3(a+b)}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{(a+b)^2}$$
$$= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{(a+2b)\cot(x)}{(a+b)^2}-\frac{\cot^3(x)}{3(a+b)}$$

Mathematica [A] time = 0.21, size = 59, normalized size = 0.97

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{\cot(x)\left((a+b)\csc^2(x)+2a+5b\right)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) - (Cot[x
]*(2*a + 5*b + (a + b)*Csc[x]^2))/(3*(a + b)^2)
```

fricas [B] time = 0.63, size = 396, normalized size = 6.49

$$\left[\frac{4\left(2a^3+7a^2b+5ab^2\right)\cos(x)^3+3\left(b^2\cos(x)^2-b^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2+4\left((2a+b)\right.}{b^2\cos(x)^4+2ab\cos(x)^2}\right.}{12\left(a^4+3a^3b+3a^2b^2+ab^3-\left(a^4+3a^3b+3a^2b^2+ab^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^3 + 3*(b^2*cos(x)^2 - b^2)*sqrt
(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)
^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b^2*
cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) - 12*(a^3 + 3*a^2*b + 2*a*b^2)*cos
(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*cos(x)^2)*sin(x)), 1/6*(2*(2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^3 + 3*(b^2
*cos(x)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(
a^2 + a*b)*cos(x)*sin(x)))*sin(x) - 6*(a^3 + 3*a^2*b + 2*a*b^2)*cos(x))/((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(
x)^2)*sin(x))]
```

giac [A] time = 0.87, size = 90, normalized size = 1.48

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{\left(a^2+2ab+b^2\right)\sqrt{a^2+ab}}-\frac{3a\tan(x)^2+6b\tan(x)^2+a+b}{3\left(a^2+2ab+b^2\right)\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^2/((a^2
+ 2*a*b + b^2)*sqrt(a^2 + a*b)) - 1/3*(3*a*tan(x)^2 + 6*b*tan(x)^2 + a + b)
/((a^2 + 2*a*b + b^2)*tan(x)^3)
```

maple [A] time = 0.12, size = 65, normalized size = 1.07

$$-\frac{1}{3(a+b)\tan(x)^3}-\frac{a}{(a+b)^2\tan(x)}-\frac{2b}{(a+b)^2\tan(x)}+\frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a+b)^2\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*cos(x)^2),x)
```

```
[Out] -1/3/(a+b)/tan(x)^3-1/(a+b)^2/tan(x)*a-2/(a+b)^2/tan(x)*b+b^2/(a+b)^2/((a+b
)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 1.09, size = 70, normalized size = 1.15

$$\frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\left(a^2+2ab+b^2\right)}-\frac{3(a+2b)\tan(x)^2+a+b}{3\left(a^2+2ab+b^2\right)\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2))
- 1/3*(3*(a + 2*b)*tan(x)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(x)^3)
```

mupad [B] time = 2.34, size = 67, normalized size = 1.10

$$\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)\left(a^2+2\,a\,b+b^2\right)}{(a+b)^{5/2}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{\frac{1}{3\,(a+b)}+\frac{\tan(x)^2\,(a+2\,b)}{(a+b)^2}}{\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + b*cos(x)^2)),x)
```

```
[Out] (b^2*atan((a^(1/2)*tan(x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(a^(1/2)*(a
+ b)^(5/2)) - (1/(3*(a + b)) + (tan(x)^2*(a + 2*b))/(a + b)^2)/tan(x)^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^4(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**4/(a + b*cos(x)**2), x)
```

3.23 $\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(a^2+3ab+3b^2\right)\cot(x)}{(a+b)^3}-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{\cot^5(x)}{5(a+b)}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}$$

```
[Out] -(a^2+3*a*b+3*b^2)*cot(x)/(a+b)^3-1/3*(2*a+3*b)*cot(x)^3/(a+b)^2-1/5*cot(x)
^5/(a+b)-b^3*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/(a+b)^(7/2)/a^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{3191, 390, 205\}$

$$-\frac{\left(a^2+3ab+3b^2\right)\cot(x)}{(a+b)^3}-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{\cot^5(x)}{5(a+b)}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^3*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2))) - ((a
^2 + 3*a*b + 3*b^2)*Cot[x])/(a + b)^3 - ((2*a + 3*b)*Cot[x]^3)/(3*(a + b)^2
) - Cot[x]^5/(5*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{a+(a+b)x^2}\,dx, x, \cot(x)\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3} + \frac{(2a+3b)x^2}{(a+b)^2} + \frac{x^4}{a+b} + \frac{b^3}{(a+b)^3\left(a+(a+b)x^2\right)}\right) dx, x, \text{c}$$

$$= -\frac{\left(a^2+3ab+3b^2\right)\cot(x)}{(a+b)^3} - \frac{(2a+3b)\cot^3(x)}{3(a+b)^2} - \frac{\cot^5(x)}{5(a+b)} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \text{c}\right.}{(a+b)^3}$$

$$= -\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}} - \frac{\left(a^2+3ab+3b^2\right)\cot(x)}{(a+b)^3} - \frac{(2a+3b)\cot^3(x)}{3(a+b)^2} - \frac{\cot^5(x)}{5(a+b)}$$

Mathematica [A] time = 0.38, size = 90, normalized size = 1.01

$$\frac{b^3\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{7/2}} - \frac{\cot(x)\left(\left(4a^2+13ab+9b^2\right)\csc^2(x)+8a^2+3(a+b)^2\csc^4(x)+26ab+33b^2\right)}{15(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(7/2)) - (Cot[x
]*(8*a^2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Csc[x]^2 + 3*(a + b)^
2*Csc[x]^4))/(15*(a + b)^3)
```

fricas [B] time = 0.86, size = 610, normalized size = 6.85

$$\left[-\frac{4\left(8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3\right)\cos(x)^5-20\left(4\,a^4+17\,a^3b+28\,a^2b^2+15\,ab^3\right)\cos(x)^3+15\left(b^3\cos(x)^4\right.}{60\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+\left(a^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^5 - 20*(4*a^4 +
 17*a^3*b + 28*a^2*b^2 + 15*a*b^3)*cos(x)^3 + 15*(b^3*cos(x)^4 - 2*b^3*cos(
x)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2
 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin
(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) + 60*(a^4 + 4*a^3*
b + 6*a^2*b^2 + 3*a*b^3)*cos(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^4 - 2*(a^5 +
 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)*sin(x)), -1/30*(2*(8*a^
4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^5 - 10*(4*a^4 + 17*a^3*b + 28*
a^2*b^2 + 15*a*b^3)*cos(x)^3 + 15*(b^3*cos(x)^4 - 2*b^3*cos(x)^2 + b^3)*sqr
t(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*si
n(x)))*sin(x) + 30*(a^4 + 4*a^3*b + 6*a^2*b^2 + 3*a*b^3)*cos(x))/((a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*
b^3 + a*b^4)*cos(x)^4 - 2*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*c
os(x)^2)*sin(x))]
```

giac [B] time = 0.60, size = 156, normalized size = 1.75

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{a^2+ab}} - \frac{15\,a^2\tan(x)^4+45\,ab\tan(x)^4+45\,b^2\tan(x)^4+10\,a^2\tan(x)^2+25\,a}{15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\text{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^3/((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) - 1/15*(15*a^2*tan(x)^4 + 45*a*
b*tan(x)^4 + 45*b^2*tan(x)^4 + 10*a^2*tan(x)^2 + 25*a*b*tan(x)^2 + 15*b^2*t
an(x)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(x)^5)
```

maple [A] time = 0.13, size = 106, normalized size = 1.19

$$-\frac{1}{5\left(a+b\right)\tan\left(x\right)^{5}}-\frac{2a}{3\left(a+b\right)^{2}\tan\left(x\right)^{3}}-\frac{b}{\left(a+b\right)^{2}\tan\left(x\right)^{3}}-\frac{a^{2}}{\left(a+b\right)^{3}\tan\left(x\right)}-\frac{3ab}{\left(a+b\right)^{3}\tan\left(x\right)}-\frac{3b^{2}}{\left(a+b\right)^{3}\tan\left(x\right)}+\frac{b^{3}\,\mathrm{arc}}{\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^6/(a+b*cos(x)^2),x)
```

```
[Out] -1/5/(a+b)/tan(x)^5-2/3/(a+b)^2/tan(x)^3*a-1/(a+b)^2/tan(x)^3*b-1/(a+b)^3/t
an(x)*a^2-3/(a+b)^3/tan(x)*a*b-3/(a+b)^3/tan(x)*b^2+b^3/(a+b)^3/((a+b)*a)^(
1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 0.85, size = 127, normalized size = 1.43

$$\frac{b^{3}\arctan\left(\frac{a\tan\left(x\right)}{\sqrt{(a+b)a}}\right)}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{(a+b)a}}-\frac{15\left(a^{2}+3\,ab+3\,b^{2}\right)\tan\left(x\right)^{4}+5\left(2\,a^{2}+5\,ab+3\,b^{2}\right)\tan\left(x\right)^{2}+3\,a^{2}+6\,ab+}{15\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\tan\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b^3*arctan(a*tan(x)/sqrt((a + b)*a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(
(a + b)*a)) - 1/15*(15*(a^2 + 3*a*b + 3*b^2)*tan(x)^4 + 5*(2*a^2 + 5*a*b +
3*b^2)*tan(x)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*t
an(x)^5)
```

mupad [B] time = 2.37, size = 101, normalized size = 1.13

$$\frac{b^{3}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan\left(x\right)\,\left(a^{3}+3\,a^{2}\,b+3\,a\,b^{2}+b^{3}\right)}{{\left(a+b\right)}^{7/2}}\right)}{\sqrt{a}\,{\left(a+b\right)}^{7/2}}-\frac{\frac{1}{5\,\left(a+b\right)}+\frac{{\tan\left(x\right)}^{2}\,\left(2\,a+3\,b\right)}{3\,{\left(a+b\right)}^{2}}+\frac{{\tan\left(x\right)}^{4}\,\left(a^{2}+3\,a\,b+3\,b^{2}\right)}{{\left(a+b\right)}^{3}}}{{\tan\left(x\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^6*(a + b*cos(x)^2)),x)
```

```
[Out] (b^3*atan((a^(1/2)*tan(x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/
(a^(1/2)*(a + b)^(7/2)) - (1/(5*(a + b)) + (tan(x)^2*(2*a + 3*b))/(3*(a + b
)^2) + (tan(x)^4*(3*a*b + a^2 + 3*b^2))/(a + b)^3)/tan(x)^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.24 $\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx$

Optimal. Leaf size=98

$$-\frac{\log\left(3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}\right)}{12\sqrt[3]{6}}+\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{6}\cos(x)+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}}$$

```
[Out] -1/12*arctan(1/3*(1+6^(1/3)*cos(x))*3^(1/2))*2^(2/3)*3^(1/6)+1/36*ln(2^(2/3
)-3^(1/3)*cos(x))*6^(2/3)-1/72*ln(2*2^(1/3)+2^(2/3)*3^(1/3)*cos(x)+3^(2/3)*
cos(x)^2)*6^(2/3)
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3223, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}\right)}{12\sqrt[3]{6}}+\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{6}\cos(x)+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(4 - 3*Cos[x]^3),x]
```

```
[Out] -ArcTan[(1 + 6^(1/3)*Cos[x])/Sqrt[3]]/(2*2^(1/3)*3^(5/6)) + Log[2^(2/3) - 3
^(1/3)*Cos[x]]/(6*6^(1/3)) - Log[2*2^(1/3) + 2^(2/3)*3^(1/3)*Cos[x] + 3^(2/
3)*Cos[x]^2]/(12*6^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = -\text{Subst}\left(\int \frac{1}{4-3x^3}\,dx, x, \cos(x)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{3}x}\,dx, x, \cos(x)\right)}{6\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{2\,2^{2/3}+\sqrt[3]{3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \cos(x)\right)}{6\sqrt[3]{2}}$$

$$= \frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \cos(x)\right)}{2\,2^{2/3}} - \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{3}+2}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}}\right.}{12}$$

$$= \frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cos(x)+3^{2/3}\cos^2(x)\right)}{12\sqrt[3]{6}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{2\sqrt[3]{}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{6}\cos(x)}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}} + \frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cos(x)+3^{2/3}\cos^2(x)\right.}{12\sqrt[3]{6}}$$

Mathematica [A] time = 0.09, size = 79, normalized size = 0.81

$$\frac{1}{72}\left(6^{2/3}\left(2\log\left(2-\sqrt[3]{6}\cos(x)\right)-\log\left(6^{2/3}\cos^2(x)+2\sqrt[3]{6}\cos(x)+4\right)\right)-6\,2^{2/3}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{6}\cos(x)+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(4 - 3*Cos[x]^3),x]
```

```
[Out] (-6*2^(2/3)*3^(1/6)*ArcTan[(1 + 6^(1/3)*Cos[x])/Sqrt[3]] + 6^(2/3)*(2*Log[2
 - 6^(1/3)*Cos[x]] - Log[4 + 2*6^(1/3)*Cos[x] + 6^(2/3)*Cos[x]^2]))/72
```

fricas [A] time = 0.57, size = 71, normalized size = 0.72

$$-\frac{1}{12}\cdot 6^{\frac{1}{6}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 6^{\frac{1}{6}}\left(6^{\frac{2}{3}}\sqrt{2}\cos(x)+6^{\frac{1}{3}}\sqrt{2}\right)\right)-\frac{1}{72}\cdot 6^{\frac{2}{3}}\log\left(-3\cos(x)^2-6^{\frac{2}{3}}\cos(x)-2\cdot 6^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 6^{\frac{2}{3}}\log\left(6^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*6^(1/6)*sqrt(2)*arctan(1/6*6^(1/6)*(6^(2/3)*sqrt(2)*cos(x) + 6^(1/3)*
sqrt(2))) - 1/72*6^(2/3)*log(-3*cos(x)^2 - 6^(2/3)*cos(x) - 2*6^(1/3)) + 1/
36*6^(2/3)*log(6^(2/3) - 3*cos(x))
```

giac [A] time = 0.20, size = 60, normalized size = 0.61

$$-\frac{1}{12}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{4}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{2}{3}}\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}+2\cos(x)\right)\right)-\frac{1}{72}\cdot 36^{\frac{1}{3}}\log\left(\cos(x)^2+\left(\frac{4}{3}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{4}{3}\right)^{\frac{2}{3}}\right)+\frac{1}{12}\left(\frac{4}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*(4/3)^(1/3)*arctan(1/4*sqrt(3)*(4/3)^(2/3)*((4/3)^(1/3) + 2*c
os(x))) - 1/72*36^(1/3)*log(cos(x)^2 + (4/3)^(1/3)*cos(x) + (4/3)^(2/3)) +
1/12*(4/3)^(1/3)*log((4/3)^(1/3) - cos(x))
```

maple [A] time = 0.05, size = 80, normalized size = 0.82

$$\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos^2(x)+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cos(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}-\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cos(x)}{2}+1\right)}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(4-3*cos(x)^3),x)
```

```
[Out] 1/36*4^(1/3)*3^(2/3)*ln(cos(x)-1/3*4^(1/3)*3^(2/3))-1/72*4^(1/3)*3^(2/3)*ln
(cos(x)^2+1/3*4^(1/3)*3^(2/3)*cos(x)+1/3*4^(2/3)*3^(1/3))-1/12*4^(1/3)*3^(1
/6)*arctan(1/3*3^(1/2)*(1/2*4^(2/3)*3^(1/3)*cos(x)+1))
```

maxima [A] time = 0.92, size = 89, normalized size = 0.91

$$-\frac{1}{72}\cdot 4^{\frac{1}{3}}3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}\cos(x)^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\cos(x)+4^{\frac{2}{3}}\right)+\frac{1}{36}\cdot 4^{\frac{1}{3}}3^{\frac{2}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\cos(x)-4^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{1}{12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="maxima")
```

```
[Out] -1/72*4^(1/3)*3^(2/3)*log(3^(2/3)*cos(x)^2 + 4^(1/3)*3^(1/3)*cos(x) + 4^(2/
3)) + 1/36*4^(1/3)*3^(2/3)*log(1/3*3^(2/3)*(3^(1/3)*cos(x) - 4^(1/3))) - 1/
12*4^(1/3)*3^(1/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*3^(2/3)*cos(x) + 4^(1/3)*
3^(1/3)))
```

mupad [B] time = 0.31, size = 75, normalized size = 0.77

$$\frac{6^{2/3}\ln\left(\cos(x)-\frac{6^{2/3}}{3}\right)}{36}+\frac{6^{2/3}\ln\left(\cos(x)-\frac{6^{2/3}\left(-1+\sqrt{3}\,1i\right)}{6}\right)\left(-1+\sqrt{3}\,1i\right)}{72}-\frac{6^{2/3}\ln\left(\cos(x)+\frac{6^{2/3}\left(1+\sqrt{3}\,1i\right)}{6}\right)\left(1+\sqrt{3}\right.}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(3*cos(x)^3 - 4),x)
```

```
[Out] (6^(2/3)*log(cos(x) - 6^(2/3)/3))/36 + (6^(2/3)*log(cos(x) - (6^(2/3)*(3^(1
/2)*1i - 1))/6)*(3^(1/2)*1i - 1))/72 - (6^(2/3)*log(cos(x) + (6^(2/3)*(3^(1
/2)*1i + 1))/6)*(3^(1/2)*1i + 1))/72
```

sympy [A] time = 1.65, size = 85, normalized size = 0.87

$$\frac{6^{\frac{2}{3}}\log\left(\cos(x)-\frac{6^{\frac{2}{3}}}{3}\right)}{36}-\frac{6^{\frac{2}{3}}\log\left(36\cos^2(x)+12\cdot 6^{\frac{2}{3}}\cos(x)+24\sqrt[3]{6}\right)}{72}-\frac{2^{\frac{2}{3}}\sqrt[6]{3}\operatorname{atan}\left(\frac{\sqrt[3]{2}\cdot 3^{\frac{5}{6}}\cos(x)}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)**3),x)
```

```
[Out] 6**(2/3)*log(cos(x) - 6**(2/3)/3)/36 - 6**(2/3)*log(36*cos(x)**2 + 12*6**(2
/3)*cos(x) + 24*6**(1/3))/72 - 2**(2/3)*3**(1/6)*atan(2**(1/3)*3**(5/6)*cos
(x)/3 + sqrt(3)/3)/12
```

3.25 $\int \frac{1}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=4

$$-\cot(x)$$

```
[Out] -cot(x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3175, 3767, 8}

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-1),x]
[Out] -Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cos^2(x)}\,dx &= \int \csc^2(x)\,dx\\ &= -\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)\\ &= -\cot(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-1),x]
[Out] -Cot[x]
```

fricas [A] time = 0.44, size = 8, normalized size = 2.00

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2),x, algorithm="fricas")
```

```
[Out] -cos(x)/sin(x)
```

giac [A] time = 0.27, size = 6, normalized size = 1.50

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/tan(x)
```

maple [A] time = 0.06, size = 7, normalized size = 1.75

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2),x)
```

```
[Out] -1/tan(x)
```

maxima [A] time = 0.50, size = 6, normalized size = 1.50

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/tan(x)
```

mupad [B] time = 2.24, size = 4, normalized size = 1.00

$$-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2 - 1),x)
```

```
[Out] -cot(x)
```

sympy [B] time = 0.43, size = 14, normalized size = 3.50

$$\frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2),x)
```

```
[Out] tan(x/2)/2 - 1/(2*tan(x/2))
```

3.26 $\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

```
[Out] -cot(x)-1/3*cot(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3175, 3767}

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-2),x]
```

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx &= \int \csc^4(x) \, dx \\
&= -\text{Subst}\left(\int \left(1+x^2\right) \, dx, x, \cot(x)\right) \\
&= -\cot(x)-\frac{\cot^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.31

$$-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-2),x]
```

```
[Out] (-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

fricas [B] time = 1.00, size = 25, normalized size = 1.92

$$-\frac{2\cos(x)^3-3\cos(x)}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

giac [A] time = 0.17, size = 14, normalized size = 1.08

$$-\frac{3 \tan(x)^2 + 1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

maple [A] time = 0.07, size = 14, normalized size = 1.08

$$-\frac{1}{3 \tan(x)^3} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^2,x)
```

```
[Out] -1/3/tan(x)^3-1/tan(x)
```

maxima [A] time = 0.87, size = 14, normalized size = 1.08

$$-\frac{3 \tan(x)^2 + 1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

mupad [B] time = 2.25, size = 10, normalized size = 0.77

$$-\frac{\cot(x) \left(\cot(x)^2 + 3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - 1)^2,x)
```

```
[Out] -(cot(x)*(cot(x)^2 + 3))/3
```

sympy [B] time = 1.14, size = 34, normalized size = 2.62

$$\frac{\tan^3\left(\frac{x}{2}\right)}{24} + \frac{3 \tan\left(\frac{x}{2}\right)}{8} - \frac{3}{8 \tan\left(\frac{x}{2}\right)} - \frac{1}{24 \tan^3\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**2,x)
```

```
[Out] tan(x/2)**3/24 + 3*tan(x/2)/8 - 3/(8*tan(x/2)) - 1/(24*tan(x/2)**3)
```

3.27 $\int \frac{1}{\left(1-\cos^2(x)\right)^3} \, dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cot^5(x) - \frac{2\cot^3(x)}{3} - \cot(x)$$

```
[Out] -cot(x)-2/3*cot(x)^3-1/5*cot(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3175, 3767}

$$-\frac{1}{5}\cot^5(x) - \frac{2\cot^3(x)}{3} - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-3),x]
```

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\cos^2(x)\right)^3} \, dx &= \int \csc^6(x) \, dx\\
&= -\text{Subst}\left(\int \left(1+2x^2+x^4\right) \, dx, x, \cot(x)\right)\\
&= -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.29

$$-\frac{8\cot(x)}{15} - \frac{1}{5}\cot(x)\csc^4(x) - \frac{4}{15}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-3),x]
```

```
[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

fricas [B] time = 0.58, size = 37, normalized size = 1.76

$$-\frac{8\cos(x)^5 - 20\cos(x)^3 + 15\cos(x)}{15\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*s
in(x))
```

giac [A] time = 0.59, size = 20, normalized size = 0.95

$$-\frac{15\tan(x)^4 + 10\tan(x)^2 + 3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

maple [A] time = 0.07, size = 20, normalized size = 0.95

$$-\frac{2}{3\tan(x)^3} - \frac{1}{5\tan(x)^5} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^3,x)
```

```
[Out] -2/3/tan(x)^3-1/5/tan(x)^5-1/tan(x)
```

maxima [A] time = 0.31, size = 20, normalized size = 0.95

$$-\frac{15\tan(x)^4 + 10\tan(x)^2 + 3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

mupad [B] time = 2.24, size = 17, normalized size = 0.81

$$-\frac{\cot(x)^5}{5} - \frac{2\cot(x)^3}{3} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2 - 1)^3,x)
```

```
[Out] - cot(x) - (2*cot(x)^3)/3 - cot(x)^5/5
```

sympy [B] time = 3.01, size = 54, normalized size = 2.57

$$\frac{\tan^5\left(\frac{x}{2}\right)}{160} + \frac{5\tan^3\left(\frac{x}{2}\right)}{96} + \frac{5\tan\left(\frac{x}{2}\right)}{16} - \frac{5}{16\tan\left(\frac{x}{2}\right)} - \frac{5}{96\tan^3\left(\frac{x}{2}\right)} - \frac{1}{160\tan^5\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**3,x)
```

```
[Out] tan(x/2)**5/160 + 5*tan(x/2)**3/96 + 5*tan(x/2)/16 - 5/(16*tan(x/2)) - 5/(9
6*tan(x/2)**3) - 1/(160*tan(x/2)**5)
```

3.28 $\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\sin(x)}{b^3}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}$$

```
[Out] (a^2-a*b+b^2)*sin(x)/b^3+1/3*(a-2*b)*sin(x)^3/b^2+1/5*sin(x)^5/b-a^3*arctan
h(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 390, 208}

$$\frac{\left(a^2-ab+b^2\right)\sin(x)}{b^3}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] -((a^3*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b])) + ((a^
2 - a*b + b^2)*Sin[x])/b^3 + ((a - 2*b)*Sin[x]^3)/(3*b^2) + Sin[x]^5/(5*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a+b-bx^2}\,dx,x,\sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{b^3}+\frac{(a-2b)x^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+b-bx^2\right)}\right)dx,x,\sin(x)\right)\\
&= \frac{\left(a^2-ab+b^2\right)\sin(x)}{b^3}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}-\frac{a^3\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\sin(x)\right)}{b^3}\\
&= -\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\sin(x)}{b^3}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 111, normalized size = 1.42

$$\frac{a^3\left(\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)-\log\left(\sqrt{a+b}+\sqrt{b}\sin(x)\right)\right)}{2b^{7/2}\sqrt{a+b}}+\frac{\left(8a^2-6ab+5b^2\right)\sin(x)}{8b^3}+\frac{(5b-4a)\sin(3x)}{48b^2}+\frac{\sin(5x)}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] (a^3*(Log[Sqrt[a + b] - Sqrt[b]*Sin[x]] - Log[Sqrt[a + b] + Sqrt[b]*Sin[x]]
))/(2*b^(7/2)*Sqrt[a + b]) + ((8*a^2 - 6*a*b + 5*b^2)*Sin[x])/(8*b^3) + ((-
4*a + 5*b)*Sin[3*x])/(48*b^2) + Sin[5*x]/(80*b)
```

fricas [A] time = 2.13, size = 259, normalized size = 3.32

$$\left[\frac{15\sqrt{ab+b^2}\,a^3\log\left(-\frac{b\cos(x)^2+2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)+2\left(3\left(ab^3+b^4\right)\cos(x)^4+15a^3b+5a^2b^2-2ab^3+8b^4-(5a\right.}{30\left(ab^4+b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a*b + b^2)*a^3*log(-(b*cos(x)^2 + 2*sqrt(a*b + b^2)*sin(x) -
 a - 2*b)/(b*cos(x)^2 + a)) + 2*(3*(a*b^3 + b^4)*cos(x)^4 + 15*a^3*b + 5*a^
2*b^2 - 2*a*b^3 + 8*b^4 - (5*a^2*b^2 + a*b^3 - 4*b^4)*cos(x)^2)*sin(x))/(a*
b^4 + b^5), 1/15*(15*sqrt(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*sin(x)/(a
 + b)) + (3*(a*b^3 + b^4)*cos(x)^4 + 15*a^3*b + 5*a^2*b^2 - 2*a*b^3 + 8*b^4
 - (5*a^2*b^2 + a*b^3 - 4*b^4)*cos(x)^2)*sin(x))/(a*b^4 + b^5)]
```

giac [A] time = 0.18, size = 96, normalized size = 1.23

$$\frac{a^3\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^3}+\frac{3b^4\sin(x)^5+5ab^3\sin(x)^3-10b^4\sin(x)^3+15a^2b^2\sin(x)-15ab^3\sin(x)+15b^4\sin(x)}{15b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] a^3*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^3) + 1/15*(3*b^4*
sin(x)^5 + 5*a*b^3*sin(x)^3 - 10*b^4*sin(x)^3 + 15*a^2*b^2*sin(x) - 15*a*b^
3*sin(x) + 15*b^4*sin(x))/b^5
```

maple [A] time = 0.06, size = 78, normalized size = 1.00

$$\frac{\frac{\left(\sin^5(x)\right)b^2}{5}+\frac{\left(\sin^3(x)\right)ab}{3}-\frac{2\left(\sin^3(x)\right)b^2}{3}+a^2\sin(x)-ab\sin(x)+b^2\sin(x)}{b^3}-\frac{a^3\operatorname{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{b^3\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a+b*cos(x)^2),x)
```

```
[Out] 1/b^3*(1/5*sin(x)^5*b^2+1/3*sin(x)^3*a*b-2/3*sin(x)^3*b^2+a^2*sin(x)-a*b*si
n(x)+b^2*sin(x))-a^3/b^3/((a+b)*b)^(1/2)*arctanh(sin(x)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 1.08, size = 91, normalized size = 1.17

$$\frac{a^3\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,b^3}+\frac{3b^2\sin(x)^5+5\left(ab-2b^2\right)\sin(x)^3+15\left(a^2-ab+b^2\right)\sin(x)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqr
t((a + b)*b)*b^3) + 1/15*(3*b^2*sin(x)^5 + 5*(a*b - 2*b^2)*sin(x)^3 + 15*(a
^2 - a*b + b^2)*sin(x))/b^3
```

mupad [B] time = 0.13, size = 86, normalized size = 1.10

$$\frac{\sin(x)^5}{5\,b} + \sin(x)^3\left(\frac{a+b}{3\,b^2} - \frac{1}{b}\right) + \sin(x)\left(\frac{3}{b} + \frac{(a+b)\left(\frac{a+b}{b^2} - \frac{3}{b}\right)}{b}\right) + \frac{a^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,1\mathrm{i}}{\sqrt{a+b}}\right)\,1\mathrm{i}}{b^{7/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a + b*cos(x)^2),x)
```

```
[Out] sin(x)^5/(5*b) + sin(x)^3*((a + b)/(3*b^2) - 1/b) + sin(x)*(3/b + ((a + b)*
((a + b)/b^2 - 3/b))/b) + (a^3*atan((b^(1/2)*sin(x)*1i)/(a + b)^(1/2))*1i)/
(b^(7/2)*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.29 $\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

```
[Out] -(a-b)*sin(x)/b^2-1/3*sin(x)^3/b+a^2*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^
(5/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 390, 208}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] (a^2*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) - ((a - b
)*Sin[x])/b^2 - Sin[x]^3/(3*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a+b-bx^2}\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2\left(a+b-bx^2\right)}\right)dx,x,\sin(x)\right)\\ &= -\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\sin(x)\right)}{b^2}\\ &= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 86, normalized size = 1.54

$$\frac{\frac{6a^2\left(\log\left(\sqrt{a+b}+\sqrt{b}\sin(x)\right)-\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)\right)}{\sqrt{a+b}}+3\sqrt{b}\,(3b-4a)\sin(x)+b^{3/2}\sin(3x)}{12b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] ((6*a^2*(-Log[Sqrt[a + b] - Sqrt[b]*Sin[x]] + Log[Sqrt[a + b] + Sqrt[b]*Sin
[x]]))/Sqrt[a + b] + 3*Sqrt[b]*(-4*a + 3*b)*Sin[x] + b^(3/2)*Sin[3*x])/(12*
b^(5/2))
```

fricas [A] time = 0.65, size = 191, normalized size = 3.41

$$\left[\frac{3\sqrt{ab+b^2}\,a^2\log\left(-\frac{b\cos(x)^2-2\sqrt{ab+b^2}\,\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)-2\left(3\,a^2b+ab^2-2\,b^3-\left(ab^2+b^3\right)\cos(x)^2\right)\sin(x)}{6\left(ab^3+b^4\right)},-\frac{3\sqrt{-ab}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a*b + b^2)*a^2*log(-(b*cos(x)^2 - 2*sqrt(a*b + b^2)*sin(x) - a
 - 2*b)/(b*cos(x)^2 + a)) - 2*(3*a^2*b + a*b^2 - 2*b^3 - (a*b^2 + b^3)*cos(
x)^2)*sin(x))/(a*b^3 + b^4), -1/3*(3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a*b
- b^2)*sin(x)/(a + b)) + (3*a^2*b + a*b^2 - 2*b^3 - (a*b^2 + b^3)*cos(x)^2)
*sin(x))/(a*b^3 + b^4)]
```

giac [A] time = 0.16, size = 65, normalized size = 1.16

$$-\frac{a^2\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^2}-\frac{b^2\sin(x)^3+3\,ab\sin(x)-3\,b^2\sin(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -a^2*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^2) - 1/3*(b^2*si
n(x)^3 + 3*a*b*sin(x) - 3*b^2*sin(x))/b^3
```

maple [A] time = 0.06, size = 50, normalized size = 0.89

$$-\frac{\frac{\left(\sin^3(x)\right)b}{3}+\sin(x)a-\sin(x)b}{b^2}+\frac{a^2\operatorname{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*cos(x)^2),x)
```

```
[Out] -1/b^2*(1/3*sin(x)^3*b+sin(x)*a-sin(x)*b)+1/b^2*a^2/((a+b)*b)^(1/2)*arctanh
(sin(x)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.69, size = 67, normalized size = 1.20

$$-\frac{a^2\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,b^2}-\frac{b\sin(x)^3+3\,(a-b)\sin(x)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sq
rt((a + b)*b)*b^2) - 1/3*(b*sin(x)^3 + 3*(a - b)*sin(x))/b^2
```

mupad [B] time = 2.31, size = 51, normalized size = 0.91

$$\frac{a^2 \operatorname{atanh}\left(\frac{\sqrt{b}\ \sin(x)}{\sqrt{a+b}}\right)}{b^{5/2} \sqrt{a+b}} - \frac{\sin(x)^3}{3\,b} - \sin(x)\left(\frac{a+b}{b^2} - \frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a + b*cos(x)^2),x)
```

```
[Out] (a^2*atanh((b^(1/2)*sin(x))/(a + b)^(1/2)))/(b^(5/2)*(a + b)^(1/2)) - sin(x
)^3/(3*b) - sin(x)*((a + b)/b^2 - 2/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{\sin(x)}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

```
[Out] sin(x)/b-a*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 388, 208}

$$\frac{\sin(x)}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sin[x]
/b
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx,x,\sin(x)\right)\\ &= \frac{\sin(x)}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\sin(x)\right)}{b}\\ &= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sin(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{\sin(x)}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sin[x]
/b
```

fricas [A] time = 0.63, size = 134, normalized size = 3.53

$$\left[\frac{\sqrt{ab+b^2}\,a\log\left(-\frac{b\cos(x)^2+2\sqrt{ab+b^2}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+2\left(ab+b^2\right)\sin(x)}{2\left(ab^2+b^3\right)}, \frac{\sqrt{-ab-b^2}\,a\arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right)+\left(ab+b^2\right)\sin(x)}{ab^2+b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*b + b^2)*a*log(-(b*cos(x)^2 + 2*sqrt(a*b + b^2)*sin(x) - a - 2
*b)/(b*cos(x)^2 + a)) + 2*(a*b + b^2)*sin(x))/(a*b^2 + b^3), (sqrt(-a*b - b
^2)*a*arctan(sqrt(-a*b - b^2)*sin(x)/(a + b)) + (a*b + b^2)*sin(x))/(a*b^2
+ b^3)]
```

giac [A] time = 0.18, size = 41, normalized size = 1.08

$$\frac{a\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b}+\frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] a*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b) + sin(x)/b
```

maple [A] time = 0.06, size = 33, normalized size = 0.87

$$\frac{\sin(x)}{b}-\frac{a\,\mathrm{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*cos(x)^2),x)
```

```
[Out] sin(x)/b-1/b*a/((a+b)*b)^(1/2)*arctanh(sin(x)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.54, size = 50, normalized size = 1.32

$$\frac{a\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,b}+\frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqrt(
(a + b)*b)*b) + sin(x)/b
```

mupad [B] time = 0.10, size = 30, normalized size = 0.79

$$\frac{\sin(x)}{b}-\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a + b*cos(x)^2),x)
```

```
[Out] sin(x)/b - (a*atanh((b^(1/2)*sin(x))/(a + b)^(1/2)))/(b^(3/2)*(a + b)^(1/2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.31 $\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

```
[Out] arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3186, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

fricas [B] time = 0.56, size = 95, normalized size = 3.28

$$\left[\frac{\log\left(-\frac{b\cos(x)^2-2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)}{2\sqrt{ab+b^2}}, -\frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right)}{ab+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(x)^2 - 2*sqrt(a*b + b^2)*sin(x) - a - 2*b)/(b*cos(x)^2 + a
))/sqrt(a*b + b^2), -sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^2)*sin(x)/(a + b
))/(a*b + b^2)]
```

giac [A] time = 0.18, size = 31, normalized size = 1.07

$$-\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(b*sin(x)/sqrt(-a*b - b^2))/sqrt(-a*b - b^2)
```

maple [A] time = 0.05, size = 21, normalized size = 0.72

$$\frac{\operatorname{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*cos(x)^2),x)
```

```
[Out] 1/((a+b)*b)^(1/2)*arctanh(sin(x)*b/((a+b)*b)^(1/2))
```

maxima [A] time = 0.85, size = 39, normalized size = 1.34

$$-\frac{\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/sqrt((a
 + b)*b)
```

mupad [B] time = 0.09, size = 21, normalized size = 0.72

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*cos(x)^2),x)
```

```
[Out] atanh((b^(1/2)*sin(x))/(a + b)^(1/2))/(b^(1/2)*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)**2),x)
[Out] Timed out
```

3.32 $\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}(\sin(x))}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

```
[Out] arctanh(sin(x))/a-arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/(a+b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3186, 391, 206, 208}

$$\frac{\tanh^{-1}(\sin(x))}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(a*Sqrt
[a + b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+b-bx^2)}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{a}$$
$$= \frac{\tanh^{-1}(\sin(x))}{a} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 0.93

$$\frac{\tanh^{-1}(\sin(x)) - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Cos[x]^2),x]
```

```
[Out] (ArcTanh[Sin[x]] - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/Sqrt[a +
 b])/a
```

fricas [A] time = 0.60, size = 119, normalized size = 2.90

$$\left[\frac{\sqrt{\frac{b}{a+b}}\log\left(-\frac{b\cos(x)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+\log(\sin(x)+1)-\log(-\sin(x)+1)}{2\,a}, \frac{2\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\sin\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log(-(b*cos(x)^2 + 2*(a + b)*sqrt(b/(a + b))*sin(x) -
 a - 2*b)/(b*cos(x)^2 + a)) + log(sin(x) + 1) - log(-sin(x) + 1))/a, 1/2*(2
*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x)) + log(sin(x) + 1) - log(-
sin(x) + 1))/a]
```

giac [A] time = 0.19, size = 57, normalized size = 1.39

$$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a} + \frac{\log(\sin(x)+1)}{2\,a} - \frac{\log(-\sin(x)+1)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a) + 1/2*log(sin(x) +
 1)/a - 1/2*log(-sin(x) + 1)/a
```

maple [A] time = 0.10, size = 47, normalized size = 1.15

$$-\frac{b\,\text{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)\,b}} - \frac{\ln(\sin(x)-1)}{2a} + \frac{\ln(\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*cos(x)^2),x)
```

```
[Out] -1/a*b/((a+b)*b)^(1/2)*arctanh(sin(x)*b/((a+b)*b)^(1/2))-1/2/a*ln(sin(x)-1)
+1/2/a*ln(sin(x)+1)
```

maxima [A] time = 0.92, size = 64, normalized size = 1.56

$$\frac{b\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,a}+\frac{\log(\sin(x)+1)}{2a}-\frac{\log(\sin(x)-1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqrt(
(a + b)*b)*a) + 1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

mupad [B] time = 2.50, size = 414, normalized size = 10.10

$$\frac{\operatorname{atanh}(\sin(x))}{a}+\frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\sin(x)+\frac{\left(2a^2b^2-\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{b(a+b)}\,1i}{a^2+ba}+\frac{\left(2b^3\sin(x)-\frac{\left(2a^2b^2+\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b}}{4\left(a^2+ba\right)}\right)}{2\left(a^2+ba\right)}\right)}{a^2+ba}}{\frac{\left(2b^3\sin(x)+\frac{\left(2a^2b^2-\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{a^2+ba}-\frac{\left(2b^3\sin(x)-\frac{\left(2a^2b^2+\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b}}{4\left(a^2+ba\right)}\right)}{2\left(a^2+ba\right)}\right)}{a^2+ba}}\right)}{a^2+ba}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*cos(x)^2)),x)
```

```
[Out] atanh(sin(x))/a + (atan((((2*b^3*sin(x) + ((2*a^2*b^2 - (sin(x)*(16*a^2*b^3
 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*
b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*sin(x) - ((2*a^2*b^2
 + (sin(x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*
(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2))/(((2*b^
3*sin(x) + ((2*a^2*b^2 - (sin(x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2)
)/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2))/(
a*b + a^2) - ((2*b^3*sin(x) - ((2*a^2*b^2 + (sin(x)*(16*a^2*b^3 + 8*a^3*b^2
)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(
b*(a + b))^(1/2))/(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)/(a + b*cos(x)**2), x)
```

3.33 $\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{(a-2b)\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a}$$

```
[Out] 1/2*(a-2*b)*arctanh(sin(x))/a^2+b^(3/2)*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))
/a^2/(a+b)^(1/2)+1/2*sec(x)*tan(x)/a
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3186, 414, 522, 206, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{(a-2b)\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Sin[x]])/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sin[x])/Sqr
t[a + b]])/(a^2*Sqrt[a + b]) + (Sec[x]*Tan[x])/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)}\,dx, x, \sin(x)\right)$$

$$= \frac{\sec(x)\tan(x)}{2a} + \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)}\,dx, x, \sin(x)\right)}{2a}$$

$$= \frac{\sec(x)\tan(x)}{2a} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{2a^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{a^2}$$

$$= \frac{(a-2b)\tanh^{-1}(\sin(x))}{2a^2} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}} + \frac{\sec(x)\tan(x)}{2a}$$

Mathematica [B] time = 0.39, size = 152, normalized size = 2.58

$$\frac{-\frac{2b^{3/2}\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)}{\sqrt{a+b}} + \frac{2b^{3/2}\log\left(\sqrt{a+b}+\sqrt{b}\sin(x)\right)}{\sqrt{a+b}} - 2(a-2b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + 2(a-2b)\log\left(\sin\left(\frac{x}{2}\right)+ \cdots\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] (-2*(a - 2*b)*Log[Cos[x/2] - Sin[x/2]] + 2*(a - 2*b)*Log[Cos[x/2] + Sin[x/2
]] - (2*b^(3/2)*Log[Sqrt[a + b] - Sqrt[b]*Sin[x]])/Sqrt[a + b] + (2*b^(3/2)
*Log[Sqrt[a + b] + Sqrt[b]*Sin[x]])/Sqrt[a + b] + a/(Cos[x/2] - Sin[x/2])^2
 - a/(Cos[x/2] + Sin[x/2])^2)/(4*a^2)
```

fricas [A] time = 0.67, size = 186, normalized size = 3.15

$$\left[\frac{2\,b\sqrt{\frac{b}{a+b}}\cos(x)^2\log\left(-\frac{b\cos(x)^2-2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right) + (a-2\,b)\cos(x)^2\log(\sin(x)+1) - (a-2\,b)\cos(x)^2\,\cdots}{4\,a^2\cos(x)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(b/(a + b))*cos(x)^2*log(-(b*cos(x)^2 - 2*(a + b)*sqrt(b/(a +
 b))*sin(x) - a - 2*b)/(b*cos(x)^2 + a)) + (a - 2*b)*cos(x)^2*log(sin(x) +
1) - (a - 2*b)*cos(x)^2*log(-sin(x) + 1) + 2*a*sin(x))/(a^2*cos(x)^2), -1/4
*(4*b*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x))*cos(x)^2 - (a - 2*b)
*cos(x)^2*log(sin(x) + 1) + (a - 2*b)*cos(x)^2*log(-sin(x) + 1) - 2*a*sin(x
))/(a^2*cos(x)^2)]
```

giac [A] time = 0.19, size = 85, normalized size = 1.44

$$-\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2} + \frac{(a-2\,b)\log(\sin(x)+1)}{4\,a^2} - \frac{(a-2\,b)\log(-\sin(x)+1)}{4\,a^2} - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2) + 1/4*(a - 2*
b)*log(sin(x) + 1)/a^2 - 1/4*(a - 2*b)*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/((
sin(x)^2 - 1)*a)
```

maple [A] time = 0.12, size = 92, normalized size = 1.56

$$\frac{b^2 \operatorname{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)\,b}} - \frac{1}{4a\left(\sin(x)-1\right)} - \frac{\ln\left(\sin(x)-1\right)}{4a} + \frac{\ln\left(\sin(x)-1\right)b}{2a^2} - \frac{1}{4a\left(\sin(x)+1\right)} + \frac{\ln\left(\sin(x)+1\right)}{4a} - \frac{\ln\left(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*cos(x)^2),x)
```

```
[Out] b^2/a^2/((a+b)*b)^(1/2)*arctanh(sin(x)*b/((a+b)*b)^(1/2))-1/4/a/(sin(x)-1)-
1/4/a*ln(sin(x)-1)+1/2/a^2*ln(sin(x)-1)*b-1/4/a/(sin(x)+1)+1/4/a*ln(sin(x)+
1)-1/2/a^2*ln(sin(x)+1)*b
```

maxima [A] time = 1.26, size = 92, normalized size = 1.56

$$-\frac{b^2 \log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,a^2} + \frac{(a-2\,b)\log\left(\sin(x)+1\right)}{4\,a^2} - \frac{(a-2\,b)\log\left(\sin(x)-1\right)}{4\,a^2} - \frac{\sin(x)}{2\left(a\sin(x)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sq
rt((a + b)*b)*a^2) + 1/4*(a - 2*b)*log(sin(x) + 1)/a^2 - 1/4*(a - 2*b)*log(
sin(x) - 1)/a^2 - 1/2*sin(x)/(a*sin(x)^2 - a)
```

mupad [B] time = 2.53, size = 483, normalized size = 8.19

$$-\frac{a^2\,\sin(x)+a^2\,\mathrm{atanh}\left(\sin(x)\right)-2\,b^2\,\mathrm{atanh}\left(\sin(x)\right)+a\,b\,\sin(x)-a\,b\,\mathrm{atanh}\left(\sin(x)\right)-a^2\,\mathrm{atanh}\left(\sin(x)\right)\,\sin(x)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a + b*cos(x)^2)),x)
```

```
[Out] -(a^2*sin(x) + a^2*atanh(sin(x)) - 2*b^2*atanh(sin(x)) + atan((b^5*sin(x)*(
a*b^3 + b^4)^(1/2)*8i - a*sin(x)*(a*b^3 + b^4)^(3/2)*4i - b*sin(x)*(a*b^3 +
 b^4)^(3/2)*8i + a*b^4*sin(x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b*sin(x)*(a*b^3
 + b^4)^(1/2)*1i + a^2*b^3*sin(x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*sin(x)*(
a*b^3 + b^4)^(1/2)*2i)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^5*b^2))*(a*b^3
+ b^4)^(1/2)*2i + a*b*sin(x) - a*b*atanh(sin(x)) - a^2*atanh(sin(x))*sin(x)
^2 + 2*b^2*atanh(sin(x))*sin(x)^2 - atan((b^5*sin(x)*(a*b^3 + b^4)^(1/2)*8i
 - a*sin(x)*(a*b^3 + b^4)^(3/2)*4i - b*sin(x)*(a*b^3 + b^4)^(3/2)*8i + a*b^
4*sin(x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b*sin(x)*(a*b^3 + b^4)^(1/2)*1i + a^
2*b^3*sin(x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*sin(x)*(a*b^3 + b^4)^(1/2)*2i
)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^5*b^2))*sin(x)^2*(a*b^3 + b^4)^(1/2)
*2i + a*b*atanh(sin(x))*sin(x)^2)/(2*a^3*sin(x)^2 - 2*a^2*b - 2*a^3 + 2*a^2
*b*sin(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**3/(a + b*cos(x)**2), x)
```

3.34 $\int \frac{\sec^5(x)}{a+b\cos^2(x)} \, dx$

Optimal. Leaf size=90

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\tan(x)\sec(x)}{8a^2}+\frac{\left(3a^2-4ab+8b^2\right)\tanh^{-1}(\sin(x))}{8a^3}+\frac{\tan(x)\sec^3(x)}{4a}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*arctanh(sin(x))/a^3-b^(5/2)*arctanh(sin(x)*b^(1/2)/
(a+b)^(1/2))/a^3/(a+b)^(1/2)+1/8*(3*a-4*b)*sec(x)*tan(x)/a^2+1/4*sec(x)^3*t
an(x)/a
```

Rubi [A] time = 0.17, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3186, 414, 527, 522, 206, 208}

$$\frac{\left(3a^2-4ab+8b^2\right)\tanh^{-1}(\sin(x))}{8a^3}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\tan(x)\sec(x)}{8a^2}+\frac{\tan(x)\sec^3(x)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Sin[x]])/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[
b]*Sin[x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]) + ((3*a - 4*b)*Sec[x]*Tan[x])/(8
*a^2) + (Sec[x]^3*Tan[x])/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+b-bx^2\right)}\,dx, x, \sin(x)\right)$$

$$= \frac{\sec^3(x)\tan(x)}{4a} + \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{\left(1-x^2\right)^2\left(a+b-bx^2\right)}\,dx, x, \sin(x)\right)}{4a}$$

$$= \frac{(3a-4b)\sec(x)\tan(x)}{8a^2} + \frac{\sec^3(x)\tan(x)}{4a} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2-(3a-4b)bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)}\,dx, x, \sin(x)\right)}{8a^2}$$

$$= \frac{(3a-4b)\sec(x)\tan(x)}{8a^2} + \frac{\sec^3(x)\tan(x)}{4a} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{a^3} + \frac{\left(3a^2-4\right.}{}$$

$$= \frac{\left(3a^2-4ab+8b^2\right)\tanh^{-1}(\sin(x))}{8a^3} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}} + \frac{(3a-4b)\sec(x)\tan(x)}{8a^2} +$$

Mathematica [B] time = 1.18, size = 215, normalized size = 2.39

$$\frac{-2\left(3a^2-4ab+8b^2\right)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\left(3a^2-4ab+8b^2\right)\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)+\frac{a^2}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^4}-}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] (-2*(3*a^2 - 4*a*b + 8*b^2)*Log[Cos[x/2] - Sin[x/2]] + 2*(3*a^2 - 4*a*b + 8
*b^2)*Log[Cos[x/2] + Sin[x/2]] + (8*b^(5/2)*Log[Sqrt[a + b] - Sqrt[b]*Sin[x
]])/Sqrt[a + b] - (8*b^(5/2)*Log[Sqrt[a + b] + Sqrt[b]*Sin[x]])/Sqrt[a + b]
 + a^2/(Cos[x/2] - Sin[x/2])^4 - a^2/(Cos[x/2] + Sin[x/2])^4 + (a*(-3*a + 4
*b))/(Cos[x/2] + Sin[x/2])^2 + (a*(-3*a + 4*b))/(-1 + Sin[x]))/(16*a^3)
```

fricas [A] time = 0.94, size = 270, normalized size = 3.00

$$\left[\frac{8\,b^2\sqrt{\frac{b}{a+b}}\cos(x)^4\log\left(-\frac{b\cos(x)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+\left(3\,a^2-4\,ab+8\,b^2\right)\cos(x)^4\log(\sin(x)+1)-\left(3\,a^2\right.}{16\,a^3\cos(x)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*b^2*sqrt(b/(a + b))*cos(x)^4*log(-(b*cos(x)^2 + 2*(a + b)*sqrt(b/(
a + b))*sin(x) - a - 2*b)/(b*cos(x)^2 + a)) + (3*a^2 - 4*a*b + 8*b^2)*cos(x
)^4*log(sin(x) + 1) - (3*a^2 - 4*a*b + 8*b^2)*cos(x)^4*log(-sin(x) + 1) + 2
*((3*a^2 - 4*a*b)*cos(x)^2 + 2*a^2)*sin(x))/(a^3*cos(x)^4), 1/16*(16*b^2*sq
rt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x))*cos(x)^4 + (3*a^2 - 4*a*b +
8*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a^2 - 4*a*b + 8*b^2)*cos(x)^4*log(-sin
(x) + 1) + 2*((3*a^2 - 4*a*b)*cos(x)^2 + 2*a^2)*sin(x))/(a^3*cos(x)^4)]
```

giac [A] time = 0.19, size = 127, normalized size = 1.41

$$\frac{b^3 \arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}+\frac{\left(3\,a^2-4\,ab+8\,b^2\right)\log\left(\sin(x)+1\right)}{16\,a^3}-\frac{\left(3\,a^2-4\,ab+8\,b^2\right)\log\left(-\sin(x)+1\right)}{16\,a^3}-\frac{3\,a\sin(x)^3-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^3) + 1/16*(3*a^2
- 4*a*b + 8*b^2)*log(sin(x) + 1)/a^3 - 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(-si
n(x) + 1)/a^3 - 1/8*(3*a*sin(x)^3 - 4*b*sin(x)^3 - 5*a*sin(x) + 4*b*sin(x))
/((sin(x)^2 - 1)^2*a^2)
```

maple [B] time = 0.13, size = 165, normalized size = 1.83

$$-\frac{b^3 \operatorname{arctanh}\left(\frac{\sin(x)b}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)\,b}}+\frac{1}{16a\left(\sin(x)-1\right)^2}-\frac{3}{16a\left(\sin(x)-1\right)}+\frac{b}{4a^2\left(\sin(x)-1\right)}-\frac{3\ln\left(\sin(x)-1\right)}{16a}+\frac{\ln\left(\sin(x)-1\right)b}{4a^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5/(a+b*cos(x)^2),x)
```

```
[Out] -b^3/a^3/((a+b)*b)^(1/2)*arctanh(sin(x)*b/((a+b)*b)^(1/2))+1/16/a/(sin(x)-1
)^2-3/16/a/(sin(x)-1)+1/4/a^2/(sin(x)-1)*b-3/16/a*ln(sin(x)-1)+1/4/a^2*ln(s
in(x)-1)*b-1/2/a^3*ln(sin(x)-1)*b^2-1/16/a/(sin(x)+1)^2-3/16/a/(sin(x)+1)+1
/4/a^2/(sin(x)+1)*b+3/16/a*ln(sin(x)+1)-1/4/a^2*ln(sin(x)+1)*b+1/2/a^3*ln(s
in(x)+1)*b^2
```

maxima [A] time = 1.99, size = 145, normalized size = 1.61

$$\frac{b^3 \log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,a^3}-\frac{(3\,a-4\,b)\sin(x)^3-(5\,a-4\,b)\sin(x)}{8\left(a^2\sin(x)^4-2\,a^2\sin(x)^2+a^2\right)}+\frac{\left(3\,a^2-4\,ab+8\,b^2\right)\log\left(\sin(x)+1\right)}{16\,a^3}-\frac{\left(3\,a^2-4\,ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqr
t((a + b)*b)*a^3) - 1/8*((3*a - 4*b)*sin(x)^3 - (5*a - 4*b)*sin(x))/(a^2*si
n(x)^4 - 2*a^2*sin(x)^2 + a^2) + 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(sin(x) +
1)/a^3 - 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(sin(x) - 1)/a^3
```

mupad [B] time = 2.64, size = 969, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5*(a + b*cos(x)^2)),x)
```

```
[Out] (5*a^3*sin(x) + atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i - a*sin(x)*(a*b^5
 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b^6*sin(x)*(a*b^5
 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i + a^2*b^5*sin(x)*(
a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)*40i + a^4*b^3*s
in(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6)^(1/2)*6i)/(40*
a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))*(a*b^5 + b^6)^(
1/2)*8i - 3*a^3*sin(x)^3 + 3*a^3*atanh(sin(x)) + 8*b^3*atanh(sin(x)) - 4*a*
b^2*sin(x) + a^2*b*sin(x) - atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i - a*s
in(x)*(a*b^5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b^6*s
in(x)*(a*b^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i + a^2*
b^5*sin(x)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)*40i
 + a^4*b^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6)^(1
/2)*6i)/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))*sin
(x)^2*(a*b^5 + b^6)^(1/2)*16i + atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i -
 a*sin(x)*(a*b^5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b
^6*sin(x)*(a*b^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i +
a^2*b^5*sin(x)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)
*40i + a^4*b^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6
)^(1/2)*6i)/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))
*sin(x)^4*(a*b^5 + b^6)^(1/2)*8i - 6*a^3*atanh(sin(x))*sin(x)^2 + 3*a^3*ata
nh(sin(x))*sin(x)^4 - 16*b^3*atanh(sin(x))*sin(x)^2 + 8*b^3*atanh(sin(x))*s
in(x)^4 + 4*a*b^2*sin(x)^3 + a^2*b*sin(x)^3 + 4*a*b^2*atanh(sin(x)) - a^2*b
*atanh(sin(x)) - 8*a*b^2*atanh(sin(x))*sin(x)^2 + 2*a^2*b*atanh(sin(x))*sin
(x)^2 + 4*a*b^2*atanh(sin(x))*sin(x)^4 - a^2*b*atanh(sin(x))*sin(x)^4)/(8*a
^4*sin(x)^4 - 16*a^4*sin(x)^2 + 8*a^3*b + 8*a^4 - 16*a^3*b*sin(x)^2 + 8*a^3
*b*sin(x)^4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.35 $\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=87

$$\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}}+\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}-\frac{(4a-3b)\sin(x)\cos(x)}{8b^2}+\frac{\sin(x)\cos^3(x)}{4b}$$

```
[Out] 1/8*(8*a^2-4*a*b+3*b^2)*x/b^3-1/8*(4*a-3*b)*cos(x)*sin(x)/b^2+1/4*cos(x)^3*
sin(x)/b+a^(5/2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/b^3/(a+b)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3187, 470, 578, 522, 203, 205}

$$\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}}-\frac{(4a-3b)\sin(x)\cos(x)}{8b^2}+\frac{\sin(x)\cos^3(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) + (a^(5/2)*ArcTan[(Sqrt[a + b]*Cot[x])/
Sqrt[a]])/(b^3*Sqrt[a + b]) - ((4*a - 3*b)*Cos[x]*Sin[x])/(8*b^2) + (Cos[x]
^3*Sin[x])/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
```

```
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)\\
&= \frac{\cos^3(x)\sin(x)}{4b} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-a+3b)x^2\right)}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)}{4b}\\
&= -\frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(x)\sin(x)}{4b} + \frac{\text{Subst}\left(\int \frac{a(4a-3b)+\left(-4a^2+ab-3b^2\right)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \cot(x\right.}{8b^2}\\
&= -\frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(x)\sin(x)}{4b} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b^3} - \frac{\left(8a^2 \right.}{}\\
&= \frac{\left(8a^2-4ab+3b^2\right)x}{8b^3} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}} - \frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(x)\sin(x}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 76, normalized size = 0.87

$$
\frac{-\frac{32a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 4x\left(8a^2-4ab+3b^2\right) - 8b(a-b)\sin(2x) + b^2\sin(4x)}{32b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (4*(8*a^2 - 4*a*b + 3*b^2)*x - (32*a^(5/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a +
 b]])/Sqrt[a + b] - 8*(a - b)*b*Sin[2*x] + b^2*Sin[4*x])/(32*b^3)
```

fricas [A] time = 1.01, size = 273, normalized size = 3.14

$$
\left[\frac{2\,a^2\sqrt{-\frac{a}{a+b}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+3\,ab\right)\cos(x)^2+4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right) + \left(8\,a^2 - \right.}{8\,b^3}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2
 + 3*a*b)*cos(x)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a^2 + a*b)*cos(x)
)*sqrt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) + (
8*a^2 - 4*a*b + 3*b^2)*x + (2*b^2*cos(x)^3 - (4*a*b - 3*b^2)*cos(x))*sin(x)
)/b^3, 1/8*(4*a^2*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(
a/(a + b))/(a*cos(x)*sin(x))) + (8*a^2 - 4*a*b + 3*b^2)*x + (2*b^2*cos(x)^3
 - (4*a*b - 3*b^2)*cos(x))*sin(x))/b^3]
```

giac [A] time = 0.19, size = 104, normalized size = 1.20

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a^3}{\sqrt{a^2+ab}\,b^3}+\frac{\left(8\,a^2-4\,ab+3\,b^2\right)x}{8\,b^3}-\frac{4\,a\tan(x)^3-3\,b\tan(x)^3+4\,a\tan(x)-5\,b\tan(x)}{8\left(\tan(x)^2+1\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a^3/(sqrt
(a^2 + a*b)*b^3) + 1/8*(8*a^2 - 4*a*b + 3*b^2)*x/b^3 - 1/8*(4*a*tan(x)^3 -
3*b*tan(x)^3 + 4*a*tan(x) - 5*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

maple [A] time = 0.07, size = 122, normalized size = 1.40

$$-\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a^3}{b^3\sqrt{(a+b)\,a}}-\frac{\left(\tan^3(x)\right)a}{2b^2\left(\tan^2(x)+1\right)^2}+\frac{3\left(\tan^3(x)\right)}{8b\left(\tan^2(x)+1\right)^2}-\frac{\tan(x)a}{2b^2\left(\tan^2(x)+1\right)^2}+\frac{5\tan(x)}{8b\left(\tan^2(x)+1\right)^2}+\frac{\arctan(\tan}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*cos(x)^2),x)
```

```
[Out] -1/b^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a^3-1/2/b^2/(tan(x)
^2+1)^2*tan(x)^3*a+3/8/b/(tan(x)^2+1)^2*tan(x)^3-1/2/b^2/(tan(x)^2+1)^2*tan
(x)*a+5/8/b/(tan(x)^2+1)^2*tan(x)+1/b^3*arctan(tan(x))*a^2-1/2/b^2*arctan(t
an(x))*a+3/8/b*arctan(tan(x))
```

maxima [A] time = 1.05, size = 97, normalized size = 1.11

$$-\frac{a^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}-\frac{(4\,a-3\,b)\tan(x)^3+(4\,a-5\,b)\tan(x)}{8\left(b^2\tan(x)^4+2\,b^2\tan(x)^2+b^2\right)}+\frac{\left(8\,a^2-4\,ab+3\,b^2\right)x}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -a^3*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3) - 1/8*((4*a - 3
*b)*tan(x)^3 + (4*a - 5*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(x)^2 + b^2) +
1/8*(8*a^2 - 4*a*b + 3*b^2)*x/b^3
```

mupad [B] time = 2.69, size = 1036, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a + b*cos(x)^2),x)
```

```
[Out] - ((tan(x)^3*(4*a - 3*b))/(8*b^2) + (tan(x)*(4*a - 5*b))/(8*b^2))/(2*tan(x)
^2 + tan(x)^4 + 1) - (atan((63*a^4*tan(x))/(64*((63*a^4)/64 - (81*a^3*b)/25
6 + (27*a^2*b^2)/256 - (35*a^5)/(32*b) + (5*a^6)/(4*b^2))) - (81*a^3*tan(x)
```

```
)/(256*((27*a^2*b)/256 - (81*a^3)/256 + (63*a^4)/(64*b) - (35*a^5)/(32*b^2)
 + (5*a^6)/(4*b^3))) - (35*a^5*tan(x))/(32*((63*a^4*b)/64 - (35*a^5)/32 + (
27*a^2*b^3)/256 - (81*a^3*b^2)/256 + (5*a^6)/(4*b))) + (5*a^6*tan(x))/(4*((
5*a^6)/4 - (35*a^5*b)/32 + (27*a^2*b^4)/256 - (81*a^3*b^3)/256 + (63*a^4*b^
2)/64)) + (27*a^2*tan(x))/(256*((27*a^2)/256 - (81*a^3)/(256*b) + (63*a^4)/
(64*b^2) - (35*a^5)/(32*b^3) + (5*a^6)/(4*b^4))))*(a^2*8i - a*b*4i + b^2*3i
)*1i)/(8*b^3) - (atan((((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^
2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(256*a^2*b^7 + 512*a^
3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) -
(tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4
))*1i)/(a*b^3 + b^4) - ((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^
2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(256*a^2*b^7 + 512*a^
3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) +
(tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4
))*1i)/(a*b^3 + b^4))/(((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^
2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(256*a^2*b^7 + 512*a^
3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) -
(tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4
)))/(a*b^3 + b^4) - ((5*a^7*b)/4 - a^8 + (9*a^5*b^3)/32 - (3*a^6*b^2)/4)/b^
6 + ((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^2*b^8)/2 - (a^3*b^7
)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(256*a^2*b^7 + 512*a^3*b^6)*(-a^5*(a + b
))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) + (tan(x)*(128*a^7 -
64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4)))/(a*b^3 + b^4)))
*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.36 $\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}+\frac{\sin(x)\cos(x)}{2b}$$

```
[Out] -1/2*(2*a-b)*x/b^2+1/2*cos(x)*sin(x)/b-a^(3/2)*arctan(cot(x)*(a+b)^(1/2)/a^
(1/2))/b^2/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3187, 470, 522, 203, 205}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}+\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -((2*a - b)*x)/(2*b^2) - (a^(3/2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(b^
2*Sqrt[a + b]) + (Cos[x]*Sin[x])/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)$$
$$= \frac{\cos(x)\sin(x)}{2b} - \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)}{2b}$$
$$= \frac{\cos(x)\sin(x)}{2b} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b^2} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(x\right.}{2b^2}$$
$$= -\frac{(2a-b)x}{2b^2} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}} + \frac{\cos(x)\sin(x)}{2b}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.87

$$\frac{\frac{4a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2x(b-2a) + b\sin(2x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (2*(-2*a + b)*x + (4*a^(3/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a +
b] + b*Sin[2*x])/(4*b^2)
```

fricas [A] time = 0.70, size = 213, normalized size = 3.55

$$\left[\frac{2\,b\cos(x)\sin(x) + a\sqrt{-\frac{a}{a+b}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+3\,ab\right)\cos(x)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\,\sin}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right.}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*cos(x)*sin(x) + a*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a
^2 + a*b)*cos(x))*sqrt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(
x)^2 + a^2)) - 2*(2*a - b)*x)/b^2, 1/2*(b*cos(x)*sin(x) - a*sqrt(a/(a + b))
*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(a/(a + b))/(a*cos(x)*sin(x))) - (
2*a - b)*x)/b^2]
```

giac [A] time = 0.22, size = 72, normalized size = 1.20

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a) + \arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}\,b^2} - \frac{(2\,a-b)x}{2\,b^2} + \frac{\tan(x)}{2\left(\tan(x)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a^2/(sqrt(
a^2 + a*b)*b^2) - 1/2*(2*a - b)*x/b^2 + 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

maple [A] time = 0.07, size = 60, normalized size = 1.00

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a^2}{b^2\sqrt{(a+b)\,a}}+\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}+\frac{\arctan\left(\tan(x)\right)}{2b}-\frac{\arctan\left(\tan(x)\right)a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*cos(x)^2),x)
```

```
[Out] 1/b^2/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a^2+1/2/b*tan(x)/(ta
n(x)^2+1)+1/2/b*arctan(tan(x))-1/b^2*arctan(tan(x))*a
```

maxima [A] time = 0.91, size = 54, normalized size = 0.90

$$\frac{a^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}-\frac{(2\,a-b)x}{2\,b^2}+\frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] a^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2) - 1/2*(2*a - b)*
x/b^2 + 1/2*tan(x)/(b*tan(x)^2 + b)
```

mupad [B] time = 2.61, size = 291, normalized size = 4.85

$$-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-b^2\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{b^2\,\sin(2\,x)}{2}+a\,b\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{a\,b\,\sin(2\,x)}{2}+\mathrm{atan}\left(\frac{a\,\sin(x)\,{\left(-a^4-b\,a^3\right)}^{3/2}\,8{}\mathrm{i}+b\,\sin(x)\,(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a + b*cos(x)^2),x)
```

```
[Out] -(2*a^2*atan(sin(x)/cos(x)) - b^2*atan(sin(x)/cos(x)) + atan((a*sin(x)*(- a
^3*b - a^4)^(3/2)*8i + b*sin(x)*(- a^3*b - a^4)^(3/2)*4i + a^5*sin(x)*(- a^
3*b - a^4)^(1/2)*8i - a^2*b^3*sin(x)*(- a^3*b - a^4)^(1/2)*2i + a^3*b^2*sin
(x)*(- a^3*b - a^4)^(1/2)*1i + a*b^4*sin(x)*(- a^3*b - a^4)^(1/2)*1i + a^4*
b*sin(x)*(- a^3*b - a^4)^(1/2)*12i)/(a^3*b^4*cos(x) - a^2*b^5*cos(x) + 5*a^
4*b^3*cos(x) + 3*a^5*b^2*cos(x)))*(- a^3*b - a^4)^(1/2)*2i - (b^2*sin(2*x))
/2 + a*b*atan(sin(x)/cos(x)) - (a*b*sin(2*x))/2)/(2*a*b^2 + 2*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.37 $\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}+\frac{x}{b}$$

```
[Out] x/b+arctan(cot(x)*(a+b)^(1/2)/a^(1/2))*a^(1/2)/b/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3171, 3181, 205}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] x/b + (Sqrt[a]*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(b*Sqrt[a + b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\cos^2(x)}\,dx}{b}\\
&= \frac{x}{b}+\frac{a\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{b}\\
&= \frac{x}{b}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.95

$$\frac{x-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a + b])/b
```

fricas [A] time = 0.62, size = 183, normalized size = 4.82

$$\left[\frac{\sqrt{-\frac{a}{a+b}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+3\,ab\right)\cos(x)^2+4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)+4\,x}{4\,b}, \frac{\sqrt{\frac{a}{a+b}}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a^2 + a*b)*cos(x))*sqrt
(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) + 4*x)/b,
 1/2*(sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(a/(a + b))/(
a*cos(x)*sin(x))) + 2*x)/b]
```

giac [A] time = 1.18, size = 48, normalized size = 1.26

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a/(sqrt(a
^2 + a*b)*b) + x/b
```

maple [A] time = 0.06, size = 32, normalized size = 0.84

$$-\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)a}{b\sqrt{(a+b)\,a}}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*cos(x)^2),x)
```

```
[Out] -1/b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))*a+x/b
```

maxima [A] time = 1.39, size = 31, normalized size = 0.82

$$-\frac{a\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) + x/b
```

mupad [B] time = 2.55, size = 425, normalized size = 11.18

$$\frac{x}{b}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2\,a^3\,\tan(x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}\,1\mathrm{i}}{b^2+a\,b}+\frac{\left(2\,a^3\,\tan(x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a\cdots}}{2\,\left(b^2+a\,b\right)}\cdots}{b^2+a\,b}}{\frac{\left(2\,a^3\,\tan(x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\tan(x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a\cdots}}{2\,\left(b^2+a\,b\right)}\cdots}{b^2+a\,b}}\right)\cdots}{b^2+a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*cos(x)^2),x)
```

```
[Out] x/b - (atan((((2*a^3*tan(x) - ((2*a^2*b^2 - (tan(x)*(8*a^2*b^3 + 16*a^3*b^2
)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))
*(-a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2*a^3*tan(x) + ((2*a^2*b^2 + (tan(x
)*(8*a^2*b^3 + 16*a^3*b^2)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b)
)^(1/2))/(2*(a*b + b^2)))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*tan(
x) - ((2*a^2*b^2 - (tan(x)*(8*a^2*b^3 + 16*a^3*b^2)*(-a*(a + b))^(1/2))/(4*
(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))*(-a*(a + b))^(1/2))/(a*b
 + b^2) - ((2*a^3*tan(x) + ((2*a^2*b^2 + (tan(x)*(8*a^2*b^3 + 16*a^3*b^2)*(
-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))*(-
a*(a + b))^(1/2))/(a*b + b^2)))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.38 $\int \frac{1}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

```
[Out] -arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = $\{3181, 205\}$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-1),x]
```

```
[Out] -(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

fricas [B] time = 0.50, size = 163, normalized size = 5.43

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{4\left(a^2+ab\right)}, -\frac{\arctan\left(\frac{(2a+b)\,\text{co}}{2\sqrt{a^2+ab}\,\text{cc}}\right.}{2\sqrt{a^2+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2*((
2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

giac [A] time = 0.38, size = 37, normalized size = 1.23

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \mathrm{sgn}(a) + \arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2 +
 a*b)
```

maple [A] time = 0.05, size = 21, normalized size = 0.70

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2),x)
```

```
[Out] 1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 1.43, size = 20, normalized size = 0.67

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

mupad [B] time = 0.00, size = 24, normalized size = 0.80

$$\frac{\mathrm{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2),x)
```

```
[Out] atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

sympy [A] time = 38.68, size = 12026, normalized size = 400.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2),x)

[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-tan(x/2)
/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (-2*tan(x/2)/(b*(tan(x/2)**2 - 1)),
Eq(a, 0)), (-5*I*a**(5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
+ b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(
a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) +
 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
 b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
+ b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + 5*I*a**(
5/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log
(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*
I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b
))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)
*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) -
 a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
+ b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*
sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 3*I*a**(5/2)*sqrt(b)*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*s
qrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) -
 a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b
))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2
*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))) + 3*I*a**(5/2)*sqrt(b)*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)))
+ 10*I*a**(3/2)*b**(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + t
an(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) +
 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
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 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(
a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
 + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 10*I*a**(3/2)*b**
(3/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(sqrt(2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7
/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/
2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
 b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
 - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))) - 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
 + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
 - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b))) + 2*I*a**(3/2)*b**(3/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*
I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)
/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/
(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - I*s
qrt(a)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*
log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/
(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(
3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sq
rt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b
/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b
**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + I*sqrt(a)*b**(5/2)*sqrt(-2
```

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*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(
a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**
2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(
a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a
/(a + b) + b/(a + b))) + I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqr
t(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sq
rt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)))
 - I*sqrt(a)*b**(5/2)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/
2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*
a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2
*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + a**3*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) -
 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
+ b/(a + b))) - a**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
 b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/
2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*
a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
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) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2
*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + a**3*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) -
 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
+ b/(a + b))) - a**3*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/
2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*
a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
 + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2
*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 10*a**2*b*sqrt(-2*I*sq
rt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*
sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sq
rt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b))) + 10*a**2*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a
**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)
*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) -
 a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)
)*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) - 2*a**2*b*sqr
t(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(-2*I*sqrt(
a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt
(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(
-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt
```

```
(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
+ b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b)
+ b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**
2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(
a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b
) - a/(a + b) + b/(a + b))) + 2*a**2*b*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a
/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b
/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
 b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
 b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(
a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b
)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I
*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/
(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(
a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b
) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b
/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + 5*a*
b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(-sqrt(2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7
/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt
(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/
2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/
(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a +
 b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
- 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*s
qrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2
*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b
)/(a + b) - a/(a + b) + b/(a + b))) - 5*a*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b))*log(sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt
(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(
a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**3*b*s
qrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*
sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*s
qrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) -
a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
+ b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))
) - 3*a*b**2*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*log(
-sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + tan(x/2))/(-8
*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a +
b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2
)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(
2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqr
t(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
 b) - a/(a + b) + b/(a + b)) - 10*a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
- a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(
a + b)) - 10*a**2*b**2*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a
 + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) + 2*a*b**3
*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a
)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))) + 3*a*b**2*sqrt(2*I*sqrt(a)*sqr
t(b)/(a + b) - a/(a + b) + b/(a + b))*log(sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
```

```
 - a/(a + b) + b/(a + b)) + tan(x/2))/(-8*I*a**(7/2)*sqrt(b)*sqrt(-2*I*sqrt
(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a +
b) - a/(a + b) + b/(a + b)) + 8*I*a**(3/2)*b**(5/2)*sqrt(-2*I*sqrt(a)*sqrt(
b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a
 + b) + b/(a + b)) + 2*a**4*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) +
 b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*
a**3*b*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*
sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b)) - 10*a**2*b**2*sqrt(-2*I*s
qrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a
 + b) - a/(a + b) + b/(a + b)) + 2*a*b**3*sqrt(-2*I*sqrt(a)*sqrt(b)/(a + b)
 - a/(a + b) + b/(a + b))*sqrt(2*I*sqrt(a)*sqrt(b)/(a + b) - a/(a + b) + b/
(a + b))), True))
```

3.39 $\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{a}$$

```
[Out] b*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(1/2)+tan(x)/a
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3187, 453, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]) + Tan[x]/a
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx&=-\text{Subst}\left(\int \frac{1+x^2}{x^2\left(a+(a+b)x^2\right)}\,dx,x,\cot(x)\right)\\&=\frac{\tan(x)}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{a}\\&=\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 38, normalized size = 1.03

$$\frac{\tan(x)}{a}-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b])) + Tan[x]/
a
```

fricas [B] time = 0.73, size = 216, normalized size = 5.84

$$\left[-\frac{\sqrt{-a^2-ab}\,b\cos(x)\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+3\,ab\right)\cos(x)^2-4\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)-4\left(a^2+\right.}{4\left(a^3+a^2b\right)\cos(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4
*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)
*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*(a^2 + a*b)*sin(x
))/((a^3 + a^2*b)*cos(x)), 1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos
(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) + 2*(a^2 + a*b)*sin(x))/
((a^3 + a^2*b)*cos(x))]
```

giac [A] time = 0.24, size = 36, normalized size = 0.97

$$-\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,a}+\frac{\tan(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(a*tan(x)/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*a) + tan(x)/a
```

maple [A] time = 0.10, size = 33, normalized size = 0.89

$$\frac{\tan(x)}{a}-\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{a\sqrt{(a+b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*cos(x)^2),x)
```

```
[Out] tan(x)/a-1/a*b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 0.89, size = 32, normalized size = 0.86

$$-\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{\tan(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a) + tan(x)/a
```

mupad [B] time = 2.38, size = 30, normalized size = 0.81

$$\frac{\tan(x)}{a} - \frac{b\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{3/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*cos(x)^2)),x)
```

```
[Out] tan(x)/a - (b*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(3/2)*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(a + b*cos(x)**2), x)
```

3.40 $\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}$$

```
[Out] -b^2*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(1/2)+(a-b)*tan(x)/a^
2+1/3*tan(x)^3/a
```

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3187, 461, 205}

$$-\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^2*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b])) + ((a -
 b)*Tan[x])/a^2 + Tan[x]^3/(3*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right) \\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{a-b}{a^2x^2}+\frac{b^2}{a^2\left(a+(a+b)x^2\right)}\right)dx, x, \cot(x)\right) \\
&= \frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{a^2} \\
&= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 55, normalized size = 0.98

$$
\frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{\tan(x)\left(a\sec^2(x)+2a-3b\right)}{3a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + ((2*a -
3*b + a*Sec[x]^2)*Tan[x])/(3*a^2)
```

fricas [B] time = 0.95, size = 276, normalized size = 4.93

$$
\left[-\frac{3\sqrt{-a^2-ab}\,b^2\cos(x)^3\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-4\left(a^3+\right.}{12\left(a^4+a^3b\right)\cos(x)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4
 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2
 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*(a^3 + a^2
*b + (2*a^3 - a^2*b - 3*a*b^2)*cos(x)^2)*sin(x))/((a^4 + a^3*b)*cos(x)^3),
-1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 +
 a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + a^2*b + (2*a^3 - a^2*b - 3*a*b^2)
*cos(x)^2)*sin(x))/((a^4 + a^3*b)*cos(x)^3)]
```

giac [A] time = 0.17, size = 71, normalized size = 1.27

$$
\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}\,a^2}+\frac{a^2\tan(x)^3+3\,a^2\tan(x)-3\,ab\tan(x)}{3\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^2/(sqrt(
a^2 + a*b)*a^2) + 1/3*(a^2*tan(x)^3 + 3*a^2*tan(x) - 3*a*b*tan(x))/a^3
```

maple [A] time = 0.11, size = 51, normalized size = 0.91

$$\frac{\tan^3(x)}{3a} + \frac{\tan(x)}{a} - \frac{\tan(x)b}{a^2} + \frac{b^2 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{a^2\sqrt{(a+b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*cos(x)^2),x)
```

```
[Out] 1/3*tan(x)^3/a+tan(x)/a-1/a^2*tan(x)*b+b^2/a^2/((a+b)*a)^(1/2)*arctan(a*tan
(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 0.87, size = 48, normalized size = 0.86

$$\frac{b^2 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2} + \frac{a\tan(x)^3 + 3\,(a-b)\tan(x)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^2) + 1/3*(a*tan(x)^
3 + 3*(a - b)*tan(x))/a^2
```

mupad [B] time = 2.31, size = 51, normalized size = 0.91

$$\frac{\tan(x)^3}{3\,a} - \tan(x)\left(\frac{a+b}{a^2} - \frac{2}{a}\right) + \frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{5/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a + b*cos(x)^2)),x)
```

```
[Out] tan(x)^3/(3*a) - tan(x)*((a + b)/a^2 - 2/a) + (b^2*atan((a^(1/2)*tan(x))/(a
 + b)^(1/2)))/(a^(5/2)*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**4/(a + b*cos(x)**2), x)
```

3.41 $\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=79

$$\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\left(a^2-ab+b^2\right)\tan(x)}{a^3}+\frac{\tan^5(x)}{5a}$$

```
[Out] b^3*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(7/2)/(a+b)^(1/2)+(a^2-a*b+b^2)*ta
n(x)/a^3+1/3*(2*a-b)*tan(x)^3/a^2+1/5*tan(x)^5/a
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3187, 461, 205}

$$\frac{\left(a^2-ab+b^2\right)\tan(x)}{a^3}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]) + ((a^2 -
a*b + b^2)*Tan[x])/a^3 + ((2*a - b)*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^6\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2a-b}{a^2x^4}+\frac{a^2-ab+b^2}{a^3x^2}+\frac{b^3}{a^3\left(-a-(a+b)x^2\right)}\right)dx, x, \cot(x)\right)$$
$$= \frac{\left(a^2-ab+b^2\right)\tan(x)}{a^3}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a}-\frac{b^3\,\text{Subst}\left(\int \frac{1}{-a-(a+b)x^2}\,dx, x, \cot(x)\right)}{a^3}$$
$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\tan(x)}{a^3}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a}$$

Mathematica [A] time = 0.35, size = 80, normalized size = 1.01

$$\frac{\tan(x)\left(3a^2\sec^4(x)+8a^2+a(4a-5b)\sec^2(x)-10ab+15b^2\right)}{15a^3}-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^3*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b])) + ((8*a
^2 - 10*a*b + 15*b^2 + a*(4*a - 5*b)*Sec[x]^2 + 3*a^2*Sec[x]^4)*Tan[x])/(15
*a^3)
```

fricas [B] time = 1.20, size = 348, normalized size = 4.41

$$\left[-\frac{15\sqrt{-a^2-ab}\,b^3\cos(x)^5\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+3ab\right)\cos(x)^2-4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-4(\ldots}{60\left(a^5+a^4b\right)\cos(\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^
2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*((8*a^4 -
 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(x)^4 + 3*a^4 + 3*a^3*b + (4*a^4 - a^3*
b - 5*a^2*b^2)*cos(x)^2)*sin(x))/((a^5 + a^4*b)*cos(x)^5), 1/30*(15*sqrt(a^
2 + a*b)*b^3*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*si
n(x)))*cos(x)^5 + 2*((8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(x)^4 + 3*
a^4 + 3*a^3*b + (4*a^4 - a^3*b - 5*a^2*b^2)*cos(x)^2)*sin(x))/((a^5 + a^4*b
)*cos(x)^5)]
```

giac [A] time = 0.18, size = 104, normalized size = 1.32

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{\sqrt{a^2+ab}\,a^3}+\frac{3a^4\tan(x)^5+10a^4\tan(x)^3-5a^3b\tan(x)^3+15a^4\tan(x)-15a^3\ldots}{15a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^3/(sqrt
(a^2 + a*b)*a^3) + 1/15*(3*a^4*tan(x)^5 + 10*a^4*tan(x)^3 - 5*a^3*b*tan(x)^
3 + 15*a^4*tan(x) - 15*a^3*b*tan(x) + 15*a^2*b^2*tan(x))/a^5
```

maple [A] time = 0.11, size = 80, normalized size = 1.01

$$\frac{\tan^5(x)}{5a}+\frac{2\left(\tan^3(x)\right)}{3a}-\frac{\left(\tan^3(x)\right)b}{3a^2}+\frac{\tan(x)}{a}-\frac{\tan(x)b}{a^2}+\frac{b^2\tan(x)}{a^3}-\frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{a^3\sqrt{(a+b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6/(a+b*cos(x)^2),x)
```

```
[Out] 1/5*tan(x)^5/a+2/3*tan(x)^3/a-1/3/a^2*tan(x)^3*b+tan(x)/a-1/a^2*tan(x)*b+1/
a^3*b^2*tan(x)-b^3/a^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

maxima [A] time = 0.86, size = 74, normalized size = 0.94

$$-\frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3}+\frac{3\,a^2\tan(x)^5+5\left(2\,a^2-ab\right)\tan(x)^3+15\left(a^2-ab+b^2\right)\tan(x)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^3) + 1/15*(3*a^2*t
an(x)^5 + 5*(2*a^2 - a*b)*tan(x)^3 + 15*(a^2 - a*b + b^2)*tan(x))/a^3
```

mupad [B] time = 2.30, size = 84, normalized size = 1.06

$$\frac{\tan(x)^5}{5\,a}-\tan(x)^3\left(\frac{a+b}{3\,a^2}-\frac{1}{a}\right)+\tan(x)\left(\frac{3}{a}+\frac{(a+b)\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right)-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{7/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6*(a + b*cos(x)^2)),x)
```

```
[Out] tan(x)^5/(5*a) - tan(x)^3*((a + b)/(3*a^2) - 1/a) + tan(x)*(3/a + ((a + b)*
((a + b)/a^2 - 3/a))/a) - (b^3*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(7/
2)*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{1}{\left(a+b\cos^2(x)\right)^2} \, dx$

Optimal. Leaf size=65

$$-\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

```
[Out] -1/2*(2*a+b)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)-1/2*b*c
os(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3184, 12, 3181, 205}

$$-\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-2),x]
```

```
[Out] -((2*a + b)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2))
 - (b*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx = -\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)} - \frac{\int \frac{-2a-b}{a+b\cos^2(x)}\,dx}{2a(a+b)}$$

$$= -\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)} + \frac{(2a+b)\int \frac{1}{a+b\cos^2(x)}\,dx}{2a(a+b)}$$

$$= -\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{2a(a+b)}$$

$$= -\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

Mathematica [**A**] time = 0.24, size = 70, normalized size = 1.08

$$-\frac{(-2a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\sin(2x)}{2a(a+b)(2a+b\cos(2x)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-2),x]
```

```
[Out] -1/2*((-2*a - b)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*(a + b)^(3/
2)) - (b*Sin[2*x])/(2*a*(a + b)*(2*a + b + b*Cos[2*x]))
```

fricas [**B**] time = 0.74, size = 326, normalized size = 5.02

$$\left[-\frac{4\left(a^2b+ab^2\right)\cos(x)\sin(x) + \left(\left(2\,ab+b^2\right)\cos(x)^2 + 2\,a^2 + ab\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4 - 2\left(4\,a^2+3\,ab\right)\cos(\cdots}{b^2\cos(x\cdots}\right.}{8\left(a^5+2\,a^4b+a^3b^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)\cos(x)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(x)*sin(x) + ((2*a*b + b^2)*cos(x)^2 + 2*a^2 +
a*b)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*
b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a
^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4
*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2), -1/4*(2*(a^2*b + a*b^2)*cos(x)*sin(x)
+ ((2*a*b + b^2)*cos(x)^2 + 2*a^2 + a*b)*sqrt(a^2 + a*b)*arctan(1/2*((2*a +
 b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^5 + 2*a^4*b + a^3*b^
2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2)]
```

giac [**A**] time = 0.18, size = 69, normalized size = 1.06

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a) + \arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(2\,a+b)}{2\left(a^2+ab\right)^{\frac{3}{2}}} - \frac{b\tan(x)}{2\left(a\tan(x)^2+a+b\right)\left(a^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(2*a +
 b)/(a^2 + a*b)^(3/2) - 1/2*b*tan(x)/((a*tan(x)^2 + a + b)*(a^2 + a*b))
```

maple [A] time = 0.07, size = 81, normalized size = 1.25

$$-\frac{b\tan(x)}{2\,(a+b)\,a\left(a\left(\tan^2(x)\right)+a+b\right)}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a+b)\sqrt{(a+b)\,a}}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)b}{2\,(a+b)\,a\sqrt{(a+b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^2,x)
```

```
[Out] -1/2*b/(a+b)/a*tan(x)/(a*tan(x)^2+a+b)+1/(a+b)/((a+b)*a)^(1/2)*arctan(a*tan
(x)/((a+b)*a)^(1/2))+1/2/(a+b)/a/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^
(1/2))*b
```

maxima [A] time = 1.09, size = 72, normalized size = 1.11

$$-\frac{b\tan(x)}{2\left(a^3+2\,a^2b+ab^2+\left(a^3+a^2b\right)\tan(x)^2\right)}+\frac{(2\,a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\left(a^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*tan(x)/(a^3 + 2*a^2*b + a*b^2 + (a^3 + a^2*b)*tan(x)^2) + 1/2*(2*a +
 b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + a*b))
```

mupad [B] time = 2.34, size = 52, normalized size = 0.80

$$\frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)(2\,a+b)}{2\,a^{3/2}\,(a+b)^{3/2}}-\frac{b\tan(x)}{2\,a\,\,(a+b)\,\left(a\tan(x)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^2,x)
```

```
[Out] (atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*(a + b)^(3/2))
- (b*tan(x))/(2*a*(a + b)*(a + b + a*tan(x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**2,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{3b(2a+b)\sin(x)\cos(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}-\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

```
[Out] -1/8*(8*a^2+8*a*b+3*b^2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(
5/2)-1/4*b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^2-3/8*b*(2*a+b)*cos(x)*sin(
x)/a^2/(a+b)^2/(a+b*cos(x)^2)
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3184, 3173, 12, 3181, 205}

$$-\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{3b(2a+b)\sin(x)\cos(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-3),x]
```

```
[Out] -((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(8*a^(5/2)*
(a + b)^(5/2)) - (b*Cos[x]*Sin[x])/(4*a*(a + b)*(a + b*Cos[x]^2)^2) - (3*b*
(2*a + b)*Cos[x]*Sin[x])/(8*a^2*(a + b)^2*(a + b*Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}-\frac{\int \frac{-4a-3b+2b\cos^2(x)}{\left(a+b\cos^2(x)\right)^2}\,dx}{4a(a+b)}$$

$$= -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}-\frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}-\frac{\int \frac{-8a^2-8ab-3b^2}{a+b\cos^2(x)}\,dx}{8a^2(a+b)^2}$$

$$= -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}-\frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}+\frac{\left(8a^2+8ab+3b^2\right)\int \frac{}{a+b}}{8a^2(a+b)^2}$$

$$= -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}-\frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}-\frac{\left(8a^2+8ab+3b^2\right)\mathrm{Subst}}{8a^2(}$$

$$= -\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}-\frac{3b(2a+b)\,\mathrm{co}}{8a^2(a+b)^2\left(a\right.}$$

Mathematica [A] time = 0.70, size = 106, normalized size = 0.99

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{\sqrt{a}\,b\sin(2x)\left(16a^2+3b(2a+b)\cos(2x)+16ab+3b^2\right)}{(a+b)^2(2a+b\cos(2x)+b)^2}}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-3),x]
```

```
[Out] (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a + b)^(5/
2) - (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 + 3*b*(2*a + b)*Cos[2*x])*Sin[2*x]
)/((a + b)^2*(2*a + b + b*Cos[2*x])^2))/(8*a^(5/2))
```

fricas [B] time = 0.77, size = 616, normalized size = 5.76

$$\left[-\frac{\left(\left(8a^2b^2+8ab^3+3b^4\right)\cos(x)^4+8a^4+8a^3b+3a^2b^2+2\left(8a^3b+8a^2b^2+3ab^3\right)\cos(x)^2\right)\sqrt{-a^2-ab}\,\log\left(\right.}{32\left(a^8+3a^7b+3a^6b^2+a^5b^3+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b
^2 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(x)^2)*sqrt(-a^2 - a*b)*log(((8*a
^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(
x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x
)^2 + a^2)) + 4*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 + (8*a^4*b + 13
*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
+ (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 + 2*(a^7*b + 3*a^6*b
^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*
cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*
```

```
cos(x)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a
*b)*cos(x)*sin(x))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 + (8*a^
4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 + 2*(a^7*b +
 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2)]
```

giac [A] time = 0.16, size = 149, normalized size = 1.39

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)\left(8\,a^2+8\,ab+3\,b^2\right)}{8\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{a^2+ab}}-\frac{8\,a^2b\tan(x)^3+5\,ab^2\tan(x)^3+8\,a^2b\tan(x)+11\,ab^2\,\mathrm{t}\cdots}{8\left(a^4+2\,a^3b+a^2b^2\right)\left(a\tan(x)^2+a+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(8*a^2
 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a^2 + a*b)) - 1/8*(8*a^2*
b*tan(x)^3 + 5*a*b^2*tan(x)^3 + 8*a^2*b*tan(x) + 11*a*b^2*tan(x) + 3*b^3*ta
n(x))/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(x)^2 + a + b)^2)
```

maple [A] time = 0.07, size = 175, normalized size = 1.64

$$\frac{-\frac{b(8a+5b)\left(\tan^3(x)\right)}{8a\left(a^2+2ab+b^2\right)}-\frac{(8a+3b)b\tan(x)}{8a^2(a+b)}}{\left(a\left(\tan^2(x)\right)+a+b\right)^2}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\left(a^2+2ab+b^2\right)\sqrt{(a+b)\,a}}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)b}{\left(a^2+2ab+b^2\right)a\sqrt{(a+b)\,a}}+\frac{3\arctan\left(\frac{a\tan(x}{\sqrt{(a+b)\cdots}}\right.}{8\left(a^2+2ab+b^2\right)a^2\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^3,x)
```

```
[Out] (-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tan(x)^3-1/8*(8*a+3*b)/a^2*b/(a+b)*tan(
x))/(a*tan(x)^2+a+b)^2+1/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)*arctan(a*tan(x)/((
a+b)*a)^(1/2))+1/(a^2+2*a*b+b^2)/a/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a
)^(1/2))*b+3/8/(a^2+2*a*b+b^2)/a^2/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a
)^(1/2))*b^2
```

maxima [A] time = 1.13, size = 186, normalized size = 1.74

$$\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{8\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{(a+b)a}}-\frac{\left(8\,a^2b+5\,ab^2\right)\tan(x)^3+\left(8\,a^2b+11\,ab^2+3\,b^3\cdots\right.}{8\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+\left(a^6+2\,a^5b+a^4b^2\right)\tan(x)^4+2\left(a^{\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*arctan(a*tan(x)/sqrt((a + b)*a))/((a^4 + 2*a^3*
b + a^2*b^2)*sqrt((a + b)*a)) - 1/8*((8*a^2*b + 5*a*b^2)*tan(x)^3 + (8*a^2*
b + 11*a*b^2 + 3*b^3)*tan(x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*
b^4 + (a^6 + 2*a^5*b + a^4*b^2)*tan(x)^4 + 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*tan(x)^2)
```

mupad [B] time = 2.44, size = 123, normalized size = 1.15

$$\frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,(a+b)^{5/2}}-\frac{\frac{\tan(x)\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,(a+b)}+\frac{\tan(x)^3\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,(a+b)^2}}{2\,a\,b+\tan(x)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\,\tan(x)^4+a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^3,x)
```

```
[Out] (atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*(
a + b)^(5/2)) - ((tan(x)*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(x)^3*(8*a*
b + 5*b^2))/(8*a*(a + b)^2))/(2*a*b + tan(x)^2*(2*a*b + 2*a^2) + a^2*tan(x)
^4 + a^2 + b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**3,x)
```

```
[Out] Timed out
```

3.44 $\int \frac{1}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=34

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 0.44

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 2.58, size = 31, normalized size = 0.91

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

giac [A] time = 0.19, size = 46, normalized size = 1.35

$$\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-\cos\left(2x\right)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1)))
```

maple [A] time = 0.05, size = 14, normalized size = 0.41

$$\frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2),x)
```

```
[Out] 1/2*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)
```

maxima [A] time = 0.82, size = 13, normalized size = 0.38

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

mupad [B] time = 2.32, size = 26, normalized size = 0.76

$$\frac{\sqrt{2}\left(x-\mathrm{atan}\left(\tan(x)\right)\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1),x)
```

```
[Out] (2^(1/2)*(x - atan(tan(x))))/2 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

sympy [A] time = 0.67, size = 63, normalized size = 1.85

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}+\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/2 + sqrt(2
)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/2
```

3.45 $\int \frac{1}{\left(1+\cos^2(x)\right)^2} dx$

Optimal. Leaf size=55

$$\frac{3x}{4\sqrt{2}} - \frac{\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)} - \frac{3\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] -1/4*cos(x)*sin(x)/(1+cos(x)^2)+3/8*x*2^(1/2)-3/8*arctan(cos(x)*sin(x)/(1+c
os(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3184, 12, 3181, 203}

$$\frac{3x}{4\sqrt{2}} - \frac{\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)} - \frac{3\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-2),x]
```

```
[Out] (3*x)/(4*Sqrt[2]) - (3*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)])/(4
*Sqrt[2]) - (Cos[x]*Sin[x])/(4*(1 + Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+\cos^2(x)\right)^2}\,dx &= -\frac{\cos(x)\sin(x)}{4\left(1+\cos^2(x)\right)}-\frac{1}{4}\int -\frac{3}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{4\left(1+\cos^2(x)\right)}+\frac{3}{4}\int \frac{1}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{4\left(1+\cos^2(x)\right)}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\ &= \frac{3x}{4\sqrt{2}}-\frac{3\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\cos(x)\sin(x)}{4\left(1+\cos^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 35, normalized size = 0.64

$$\frac{3\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\sin(2x)}{4(\cos(2x)+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-2),x]
[Out] (3*ArcTan[Tan[x]/Sqrt[2]])/(4*Sqrt[2]) - Sin[2*x]/(4*(3 + Cos[2*x]))
```

fricas [A] time = 0.62, size = 57, normalized size = 1.04

$$-\frac{3\left(\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4\cos(x)\sin(x)}{16\left(\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^2,x, algorithm="fricas")
[Out] -1/16*(3*(sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt
(2))/(cos(x)*sin(x))) + 4*cos(x)*sin(x))/(cos(x)^2 + 1)
```

giac [A] time = 1.29, size = 59, normalized size = 1.07

$$\frac{3}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-\frac{\tan(x)}{4\left(\tan(x)^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^2,x, algorithm="giac")
[Out] 3/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - 1/4*tan(x)/(tan(x)^2 + 2)
```

maple [A] time = 0.06, size = 27, normalized size = 0.49

$$-\frac{\tan(x)}{4\left(\tan^2(x)+2\right)}+\frac{3\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^2,x)
```

```
[Out] -1/4*tan(x)/(tan(x)^2+2)+3/8*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)
```

maxima [A] time = 0.96, size = 26, normalized size = 0.47

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-\frac{\tan(x)}{4\left(\tan(x)^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/4*tan(x)/(tan(x)^2 + 2)
```

mupad [B] time = 2.17, size = 40, normalized size = 0.73

$$\frac{3\sqrt{2}\,\left(x-\mathrm{atan}\left(\tan(x)\right)\right)}{8}-\frac{\tan(x)}{4\left(\tan(x)^2+2\right)}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^2,x)
```

```
[Out] (3*2^(1/2)*(x - atan(tan(x))))/8 - tan(x)/(4*(tan(x)^2 + 2)) + (3*2^(1/2)*a
tan((2^(1/2)*tan(x))/2))/8
```

sympy [B] time = 3.44, size = 218, normalized size = 3.96

$$\frac{3\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+8}+\frac{3\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{8\tan^4\left(\frac{x}{2}\right)+8}+\frac{3\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\,\mathrm{ta}\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**2,x)
```

```
[Out] 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)
**4/(8*tan(x/2)**4 + 8) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor(
(x/2 - pi/2)/pi))/(8*tan(x/2)**4 + 8) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) +
1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(8*tan(x/2)**4 + 8) + 3*sqrt(2)
*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(8*tan(x/2)**4 +
8) + 2*tan(x/2)**3/(8*tan(x/2)**4 + 8) - 2*tan(x/2)/(8*tan(x/2)**4 + 8)
```

3.46 $\int \frac{1}{\left(1+\cos^2(x)\right)^3} dx$

Optimal. Leaf size=71

$$\frac{19x}{32\sqrt{2}} - \frac{9\sin(x)\cos(x)}{32\left(\cos^2(x)+1\right)} - \frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2} - \frac{19\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{32\sqrt{2}}$$

```
[Out] -1/8*cos(x)*sin(x)/(1+cos(x)^2)^2-9/32*cos(x)*sin(x)/(1+cos(x)^2)+19/64*x*2
^(1/2)-19/64*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3184, 3173, 12, 3181, 203}

$$\frac{19x}{32\sqrt{2}} - \frac{9\sin(x)\cos(x)}{32\left(\cos^2(x)+1\right)} - \frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2} - \frac{19\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-3),x]
```

```
[Out] (19*x)/(32*Sqrt[2]) - (19*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)])
/(32*Sqrt[2]) - (Cos[x]*Sin[x])/(8*(1 + Cos[x]^2)^2) - (9*Cos[x]*Sin[x])/(3
2*(1 + Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+\cos^2(x)\right)^3}\,dx &= -\frac{\cos(x)\sin(x)}{8\left(1+\cos^2(x)\right)^2}-\frac{1}{8}\int \frac{-7+2\cos^2(x)}{\left(1+\cos^2(x)\right)^2}\,dx\\ &= -\frac{\cos(x)\sin(x)}{8\left(1+\cos^2(x)\right)^2}-\frac{9\cos(x)\sin(x)}{32\left(1+\cos^2(x)\right)}-\frac{1}{32}\int -\frac{19}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{8\left(1+\cos^2(x)\right)^2}-\frac{9\cos(x)\sin(x)}{32\left(1+\cos^2(x)\right)}+\frac{19}{32}\int \frac{1}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{8\left(1+\cos^2(x)\right)^2}-\frac{9\cos(x)\sin(x)}{32\left(1+\cos^2(x)\right)}-\frac{19}{32}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\ &= \frac{19x}{32\sqrt{2}}-\frac{19\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{32\sqrt{2}}-\frac{\cos(x)\sin(x)}{8\left(1+\cos^2(x)\right)^2}-\frac{9\cos(x)\sin(x)}{32\left(1+\cos^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 51, normalized size = 0.72

$$\frac{19\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{9\sin(2x)}{32(\cos(2x)+3)}-\frac{\sin(2x)}{4(\cos(2x)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-3),x]
```

```
[Out] (19*ArcTan[Tan[x]/Sqrt[2]])/(32*Sqrt[2]) - Sin[2*x]/(4*(3 + Cos[2*x])^2) -
(9*Sin[2*x])/(32*(3 + Cos[2*x]))
```

fricas [A] time = 1.75, size = 81, normalized size = 1.14

$$-\frac{19\left(\sqrt{2}\cos(x)^4+2\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4\left(9\cos(x)^3+13\cos(x)\right)\sin(x)}{128\left(\cos(x)^4+2\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(19*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*
sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) + 4*(9*cos(x)^3 + 13*cos(x))*s
in(x))/(cos(x)^4 + 2*cos(x)^2 + 1)
```

giac [A] time = 0.15, size = 68, normalized size = 0.96

$$\frac{19}{64}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-\frac{13\tan(x)^3+22\tan(x)}{32\left(\tan(x)^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^3,x, algorithm="giac")
```

```
[Out] 19/64*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - cos(2*x) + 1))) - 1/32*(13*tan(x)^3 + 22*tan(x))/(tan(x)^2 + 2)
^2
```

maple [A] time = 0.06, size = 35, normalized size = 0.49

$$\frac{-\frac{13(\tan^3(x))}{32}-\frac{11\tan(x)}{16}}{\left(\tan^2(x)+2\right)^2}+\frac{19\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^3,x)
```

```
[Out] (-13/32*tan(x)^3-11/16*tan(x))/(tan(x)^2+2)^2+19/64*arctan(1/2*tan(x)*2^(1/
2))*2^(1/2)
```

maxima [A] time = 1.57, size = 41, normalized size = 0.58

$$\frac{19}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-\frac{13\tan(x)^3+22\tan(x)}{32\left(\tan(x)^4+4\tan(x)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 19/64*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/32*(13*tan(x)^3 + 22*tan(x))/(
tan(x)^4 + 4*tan(x)^2 + 4)
```

mupad [B] time = 2.15, size = 53, normalized size = 0.75

$$\frac{19\sqrt{2}\,\left(x-\mathrm{atan}\left(\tan(x)\right)\right)}{64}-\frac{\frac{13\tan(x)^3}{32}+\frac{11\tan(x)}{16}}{\tan(x)^4+4\tan(x)^2+4}+\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^3,x)
```

```
[Out] (19*2^(1/2)*(x - atan(tan(x))))/64 - ((11*tan(x))/16 + (13*tan(x)^3)/32)/(4
*tan(x)^2 + tan(x)^4 + 4) + (19*2^(1/2)*atan((2^(1/2)*tan(x))/2))/64
```

sympy [B] time = 13.61, size = 439, normalized size = 6.18

$$\frac{19\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^8\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64}+\frac{38\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64}+\frac{19\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**3,x)
```

```
[Out] 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2
)**8/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 38*sqrt(2)*(atan(sqrt(2)*tan
(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(64*tan(x/2)**8 + 128*t
an(x/2)**4 + 64) + 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 -
 pi/2)/pi))/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 19*sqrt(2)*(atan(sqrt
(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**8/(64*tan(x/2)**8
+ 128*tan(x/2)**4 + 64) + 38*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor
((x/2 - pi/2)/pi))*tan(x/2)**4/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 19
*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(64*tan(x
/2)**8 + 128*tan(x/2)**4 + 64) + 22*tan(x/2)**7/(64*tan(x/2)**8 + 128*tan(x
/2)**4 + 64) - 14*tan(x/2)**5/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 14*
tan(x/2)**3/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) - 22*tan(x/2)/(64*tan(x
/2)**8 + 128*tan(x/2)**4 + 64)
```

3.47 $\int \sqrt{1-\cos^2(x)}\,dx$

Optimal. Leaf size=12

$$\sqrt{\sin^2(x)}\,(-\cot(x))$$

```
[Out] -cot(x)*(sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3176, 3207, 2638}

$$\sqrt{\sin^2(x)}\,(-\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[Sin[x]^2])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\cos^2(x)}\,dx &= \int \sqrt{\sin^2(x)}\,dx\\
&= \left(\csc(x)\sqrt{\sin^2(x)}\right)\int \sin(x)\,dx\\
&= -\cot(x)\sqrt{\sin^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\sqrt{\sin^2(x)}\,(-\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[Sin[x]^2])
```

fricas [A] time = 0.69, size = 4, normalized size = 0.33

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] -cos(x)
```

giac [B] time = 1.71, size = 24, normalized size = 2.00

$$-\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{3}+\tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2),x, algorithm="giac")
[Out] -2*sgn(tan(1/2*x)^3 + tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

maple [A] time = 0.86, size = 13, normalized size = 1.08

$$-\frac{2\sin(x)\cos(x)}{\sqrt{2-2\cos(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)^(1/2),x)
[Out] -sin(x)*cos(x)/(sin(x)^2)^(1/2)
```

maxima [A] time = 0.89, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\tan(x)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(tan(x)^2 + 1)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.83

$$-\cot(x)\sqrt{\sin(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x)^2)^(1/2),x)
[Out] -cot(x)*(sin(x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{1-\cos^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)**(1/2),x)
[Out] Integral(sqrt(1 - cos(x)**2), x)
```

3.48 $\int \sqrt{-1+\cos^2(x)}\,dx$

Optimal. Leaf size=14

$$\sqrt{-\sin^2(x)}\,(-\cot(x))$$

```
[Out] -cot(x)*(-sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3176, 3207, 2638}

$$\sqrt{-\sin^2(x)}\,(-\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Cos[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[-Sin[x]^2])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-1+\cos^2(x)}\,dx &= \int \sqrt{-\sin^2(x)}\,dx\\
&= \left(\csc(x)\sqrt{-\sin^2(x)}\right)\int \sin(x)\,dx\\
&= -\cot(x)\sqrt{-\sin^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\sqrt{-\sin^2(x)}\,(-\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Cos[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[-Sin[x]^2])
```

fricas [A] time = 1.47, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 1.25, size = 28, normalized size = 2.00

$$\frac{2i\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{3}-\tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*I*sgn(-tan(1/2*x)^3 - tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

maple [A] time = 0.54, size = 14, normalized size = 1.00

$$\frac{\sin(x)\cos(x)}{\sqrt{-\left(\sin^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cos(x)^2)^(1/2),x)
[Out] sin(x)*cos(x)/(-sin(x)^2)^(1/2)
```

maxima [A] time = 0.88, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{-\tan(x)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(-tan(x)^2 - 1)
```

mupad [B] time = 2.29, size = 39, normalized size = 2.79

$$-\frac{\sqrt{-4\sin(x)^{2}}\,\left(-\sin(x)^{2}+\frac{\sin(2\,x)\,1\mathrm{i}}{2}+1\right)}{\sin(x)^{2}\,2\mathrm{i}+\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 - 1)^(1/2),x)
[Out] -((-4*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(sin(2*x) + sin(x)^
2*2i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cos^{2}\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x)**2 - 1), x)
```

3.49 $\int \left(1-\cos^2(x)\right)^{3/2} dx$

Optimal. Leaf size=29

$$-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)-\frac{2}{3}\sqrt{\sin^2(x)}\cot(x)$$

```
[Out] -1/3*cot(x)*(sin(x)^2)^(3/2)-2/3*cot(x)*(sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3176, 3203, 3207, 2638}

$$-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)-\frac{2}{3}\sqrt{\sin^2(x)}\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(3/2),x]
```

```
[Out] (-2*Cot[x]*Sqrt[Sin[x]^2])/3 - (Cot[x]*(Sin[x]^2)^(3/2))/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(1-\cos^2(x)\right)^{3/2} dx &= \int \sin^2(x)^{3/2}\,dx\\
&= -\frac{1}{3}\cot(x)\sin^2(x)^{3/2}+\frac{2}{3}\int \sqrt{\sin^2(x)}\,dx\\
&= -\frac{1}{3}\cot(x)\sin^2(x)^{3/2}+\frac{1}{3}\left(2\csc(x)\sqrt{\sin^2(x)}\right)\int \sin(x)\,dx\\
&= -\frac{2}{3}\cot(x)\sqrt{\sin^2(x)}-\frac{1}{3}\cot(x)\sin^2(x)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.79

$$\frac{1}{12}\sqrt{\sin^2(x)}\,(\cos(3x)-9\cos(x))\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(3/2),x]
```

```
[Out] ((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[Sin[x]^2])/12
```

fricas [A] time = 0.68, size = 11, normalized size = 0.38

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

giac [B] time = 0.23, size = 45, normalized size = 1.55

$$-\frac{4\left(3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2+\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -4/3*(3*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2 + sgn(tan(1/2*x)^3 + ta
n(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

maple [A] time = 1.43, size = 19, normalized size = 0.66

$$\frac{2\sin(x)\cos(x)\left(\cos^2(x)-3\right)}{3\sqrt{2-2\cos(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)^(3/2),x)
```

```
[Out] 1/3*sin(x)*cos(x)*(cos(x)^2-3)/(sin(x)^2)^(1/2)
```

maxima [A] time = 0.98, size = 11, normalized size = 0.38

$$-\frac{1}{12}\cos(3x)+\frac{3}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*cos(3*x) + 3/4*cos(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(1-\cos(x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x)^2)^(3/2),x)
```

```
[Out] int((1 - cos(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\cos ^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - cos(x)**2)**(3/2), x)
```

3.50 $\int \left(-1+\cos^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=33

$$\frac{2}{3}\sqrt{-\sin^2(x)}\,\cot(x)-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

```
[Out] -1/3*cot(x)*(-sin(x)^2)^(3/2)+2/3*cot(x)*(-sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3176, 3203, 3207, 2638}

$$\frac{2}{3}\sqrt{-\sin^2(x)}\,\cot(x)-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Cos[x]^2)^(3/2),x]
```

```
[Out] (2*Cot[x]*Sqrt[-Sin[x]^2])/3 - (Cot[x]*(-Sin[x]^2)^(3/2))/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(-1+\cos^2(x)\right)^{3/2} dx &= \int \left(-\sin^2(x)\right)^{3/2} dx\\
&= -\frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2}-\frac{2}{3}\int \sqrt{-\sin^2(x)}\,dx\\
&= -\frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2}-\frac{1}{3}\left(2\csc(x)\sqrt{-\sin^2(x)}\right)\int \sin(x)\,dx\\
&= \frac{2}{3}\cot(x)\sqrt{-\sin^2(x)}-\frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.76

$$-\frac{1}{12}\sqrt{-\sin^2(x)}\,(\cos(3x)-9\cos(x))\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cos[x]^2)^(3/2),x]
[Out] -1/12*((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[-Sin[x]^2])
```

fricas [A] time = 0.45, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.65, size = 55, normalized size = 1.67

$$-\frac{12i\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2+4i\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/3*(12*I*sgn(-tan(1/2*x)^3 - tan(1/2*x))*tan(1/2*x)^2 + 4*I*sgn(-tan(1/2*
x)^3 - tan(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

maple [A] time = 0.99, size = 21, normalized size = 0.64

$$-\frac{\sin(x)\cos(x)\left(\sin^2(x)+2\right)}{3\sqrt{-\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cos(x)^2)^(3/2),x)
[Out] -1/3*sin(x)*cos(x)*(sin(x)^2+2)/(-sin(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\cos(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((cos(x)^2 - 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(\cos(x)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int((cos(x)^2 - 1)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cos^2(x) - 1\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 - 1)**(3/2), x)
```

3.51 $\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$

Optimal. Leaf size=15

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{\sin^2(x)}}$$

```
[Out] -arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3176, 3207, 3770}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cos[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[Sin[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx &= \int \frac{1}{\sqrt{\sin^2(x)}}\,dx\\ &= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{\sin^2(x)}}\\ &= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{\sin^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.87

$$\frac{\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cos[x]^2],x]
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[Sin[x]^2]
```

fricas [A] time = 1.35, size = 19, normalized size = 1.27

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-cos(x)^2 + 1), x)
```

maple [A] time = 0.71, size = 14, normalized size = 0.93

$$-\frac{2\,\mathrm{arctanh}\left(\cos(x)\right)\sin(x)}{\sqrt{2-2\cos\left(2x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^(1/2),x)
[Out] -arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

maxima [B] time = 1.13, size = 35, normalized size = 2.33

$$\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)-\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*log(cos(x)^2 + sin(x)^2 -
 2*cos(x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{1-\cos(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cos(x)^2)^(1/2),x)
[Out] int(1/(1 - cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cos(x)**2), x)
```

3.52 $\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{-\sin^2(x)}}$$

```
[Out] -arctanh(cos(x))*sin(x)/(-sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3176, 3207, 3770}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Cos[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[-Sin[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx &= \int \frac{1}{\sqrt{-\sin^2(x)}}\,dx\\
&= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{-\sin^2(x)}}\\
&= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{-\sin^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.76

$$\frac{\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Cos[x]^2],x]
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[-Sin[x]^2]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(x)^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(cos(x)^2 - 1), x)
```

maple [B] time = 0.41, size = 34, normalized size = 2.00

$$-\frac{\sin(x)\sqrt{-\left(\cos^2(x)\right)}\ \arctan\left(\frac{1}{\sqrt{-\left(\cos^2(x)\right)}}\right)}{\cos(x)\sqrt{-\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cos(x)^2)^(1/2),x)
[Out] -sin(x)*(-cos(x)^2)^(1/2)*arctan(1/(-cos(x)^2)^(1/2))/cos(x)/(-sin(x)^2)^(1
/2)
```

maxima [A] time = 0.90, size = 17, normalized size = 1.00

$$-\arctan\left(\sin(x), \cos(x) + 1\right) + \arctan\left(\sin(x), \cos(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -arctan2(sin(x), cos(x) + 1) + arctan2(sin(x), cos(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{{\cos(x)}^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - 1)^(1/2),x)
[Out] int(1/(cos(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos^2(x) - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(x)**2 - 1), x)
```

3.53 $\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2\sqrt{\sin^2(x)}}$$

```
[Out] -1/2*cot(x)/(sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3176, 3204, 3207, 3770}

$$-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-3/2),x]
```

```
[Out] -Cot[x]/(2*Sqrt[Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[Sin[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\sin^2(x)^{3/2}}\,dx$$
$$= -\frac{\cot(x)}{2\sqrt{\sin^2(x)}} + \frac{1}{2}\int \frac{1}{\sqrt{\sin^2(x)}}\,dx$$
$$= -\frac{\cot(x)}{2\sqrt{\sin^2(x)}} + \frac{\sin(x)\int \csc(x)\,dx}{2\sqrt{\sin^2(x)}}$$
$$= -\frac{\cot(x)}{2\sqrt{\sin^2(x)}} - \frac{\tanh^{-1}(\cos(x))\sin(x)}{2\sqrt{\sin^2(x)}}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 1.59

$$-\frac{\sin(x)\left(\csc^2\left(\frac{x}{2}\right)-\sec^2\left(\frac{x}{2}\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{8\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-3/2),x]
[Out] -1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x])
/Sqrt[Sin[x]^2]
```

fricas [A] time = 0.60, size = 44, normalized size = 1.38

$$-\frac{\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

giac [B] time = 0.26, size = 78, normalized size = 2.44

$$\frac{\tan\left(\frac{1}{2}x\right)^2}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)}+\frac{\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{4\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)}-\frac{2\tan\left(\frac{1}{2}x\right)^2+1}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/8*tan(1/2*x)^2/sgn(tan(1/2*x)^3 + tan(1/2*x)) + 1/4*log(tan(1/2*x)^2)/sgn
(tan(1/2*x)^3 + tan(1/2*x)) - 1/8*(2*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x)^3 +
tan(1/2*x))*tan(1/2*x)^2)
```

maple [A] time = 1.56, size = 37, normalized size = 1.16

$$-\frac{2\left(\frac{\cos(x)}{2}+\frac{\left(-\ln(-1+\cos(x))+\ln(\cos(x)+1)\right)\left(\sin^2(x)\right)}{4}\right)}{\sin(x)\sqrt{2-2\cos(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^(3/2),x)
```

```
[Out] -(1/2*cos(x)+1/4*(-ln(-1+cos(x))+ln(cos(x)+1))*sin(x)^2)/sin(x)/(sin(x)^2)^
(1/2)
```

maxima [B] time = 1.84, size = 300, normalized size = 9.38

$$\frac{4\left(\cos\left(3\,x\right)+\cos\left(x\right)\right)\cos\left(4\,x\right)-4\left(2\,\cos\left(2\,x\right)-1\right)\cos\left(3\,x\right)-8\,\cos\left(2\,x\right)\cos\left(x\right)+\left(2\left(2\,\cos\left(2\,x\right)-1\right)\cos\left(4\,x\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(2
*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin
(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^2
 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4
*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
- 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4*
x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(2*x) - 1
)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) -
 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(1-{\cos\left(x\right)}^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(1 - cos(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1 - \cos^{2}{\left(x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - cos(x)**2)**(-3/2), x)
```

3.54 $\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\, dx$

Optimal. Leaf size=36

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}+\frac{\sin(x)\tanh^{-1}(\cos(x))}{2\sqrt{-\sin^2(x)}}$$

```
[Out] 1/2*cot(x)/(-sin(x)^2)^(1/2)+1/2*arctanh(cos(x))*sin(x)/(-sin(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3176, 3204, 3207, 3770}

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}+\frac{\sin(x)\tanh^{-1}(\cos(x))}{2\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] Cot[x]/(2*Sqrt[-Sin[x]^2]) + (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[-Sin[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-\sin^2(x)\right)^{3/2}}\,dx$$
$$= \frac{\cot(x)}{2\sqrt{-\sin^2(x)}} - \frac{1}{2}\int \frac{1}{\sqrt{-\sin^2(x)}}\,dx$$
$$= \frac{\cot(x)}{2\sqrt{-\sin^2(x)}} - \frac{\sin(x)\int \csc(x)\,dx}{2\sqrt{-\sin^2(x)}}$$
$$= \frac{\cot(x)}{2\sqrt{-\sin^2(x)}} + \frac{\tanh^{-1}(\cos(x))\sin(x)}{2\sqrt{-\sin^2(x)}}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.47

$$\frac{\sin(x)\left(\csc^2\left(\frac{x}{2}\right)-\sec^2\left(\frac{x}{2}\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{8\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] ((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x])/(8*S
qrt[-Sin[x]^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [C] time = 0.69, size = 90, normalized size = 2.50

$$-\frac{i\tan\left(\frac{1}{2}x\right)^2}{8\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)}-\frac{i\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)}+\frac{2i\tan\left(\frac{1}{2}x\right)^2+i}{8\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*I*tan(1/2*x)^2/sgn(-tan(1/2*x)^3 - tan(1/2*x)) - 1/4*I*log(tan(1/2*x)^
2)/sgn(-tan(1/2*x)^3 - tan(1/2*x)) + 1/8*(2*I*tan(1/2*x)^2 + I)/(sgn(-tan(1
/2*x)^3 - tan(1/2*x))*tan(1/2*x)^2)
```

maple [A] time = 1.18, size = 51, normalized size = 1.42

$$-\frac{\sqrt{-\left(\cos^2(x)\right)}\left(-\arctan\left(\frac{1}{\sqrt{-\left(\cos^2(x)\right)}}\right)\left(\sin^2(x)\right)+\sqrt{-\left(\cos^2(x)\right)}\right)}{2\sin(x)\cos(x)\sqrt{-\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cos(x)^2)^(3/2),x)
```

```
[Out] -1/2/sin(x)*(-cos(x)^2)^(1/2)*(-arctan(1/(-cos(x)^2)^(1/2))*sin(x)^2+(-cos(
x)^2)^(1/2))/cos(x)/(-sin(x)^2)^(1/2)
```

maxima [B] time = 1.05, size = 284, normalized size = 7.89

$$\left(2\,(2\,\cos(2\,x)-1)\cos(4\,x)-\cos(4\,x)^2-4\,\cos(2\,x)^2-\sin(4\,x)^2+4\,\sin(4\,x)\sin(2\,x)-4\,\sin(2\,x)^2+4\,\cos(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
+ 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos(
x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*
x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x),
 cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(4
*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x)
 - 4*cos(2*x)*sin(x) + 2*sin(x))/(2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2
- 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*
x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(\cos(x)^2-1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int(1/(cos(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos^2(x)-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 - 1)**(-3/2), x)
```

3.55 $\int \sqrt{1+\cos^2(x)}\,dx$

Optimal. Leaf size=9

$$E\left(x+\frac{\pi}{2}\middle|-1\right)$$

[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3177}

$$E\left(x+\frac{\pi}{2}\middle|-1\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Cos[x]^2],x]

[Out] EllipticE[Pi/2 + x, -1]

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{1+\cos^2(x)}\,dx = E\left(\frac{\pi}{2}+x\middle|-1\right)$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.22

$$\sqrt{2}\,E\left(x\middle|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Cos[x]^2],x]

[Out] Sqrt[2]*EllipticE[x, 1/2]

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\cos(x)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(cos(x)^2 + 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(x)^2 + 1), x)

maple [B] time = 1.56, size = 41, normalized size = 4.56

$$-\frac{\sqrt{\left(1+\cos^{2}(x)\right)\left(\sin^{2}(x)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\operatorname{EllipticE}\left(\cos(x),i\right)}{\sqrt{1-\left(\cos^{4}(x)\right)}\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)^(1/2),x)
```

```
[Out] -((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)*EllipticE(cos(x),I)/(1-cos(
x)^4)^(1/2)/sin(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cos(x)^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(x)^2 + 1), x)
```

mupad [B] time = 0.01, size = 7, normalized size = 0.78

$$\sqrt{2}\,\mathrm{E}\left(x\,\middle|\,\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*ellipticE(x, 1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cos^{2}{\left(x\right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x)**2 + 1), x)
```

3.56 $\int \sqrt{-1-\cos^2(x)}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^
2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3178, 3177}

$$\frac{\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] (Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\cos^2(x)}\,dx &= \frac{\sqrt{-1-\cos^2(x)}\int \sqrt{1+\cos^2(x)}\,dx}{\sqrt{1+\cos^2(x)}}\\ &= \frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.06

$$-\frac{\sqrt{2}\sqrt{\cos(2x)+3}\,E\left(x\middle|\frac{1}{2}\right)}{\sqrt{-\cos(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] -((Sqrt[2]*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2])/Sqrt[-3 - Cos[2*x]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\frac{2\left(e^{(2ix)}-e^{(ix)}\right)\mathrm{integral}\left(\frac{4\sqrt{e^{(4ix)}+6e^{(2ix)}+1}\left(e^{(2ix)}+1\right)}{e^{(6ix)}-2e^{(5ix)}+7e^{(4ix)}-12e^{(3ix)}+7e^{(2ix)}-2e^{(ix)}+1},x\right)+\sqrt{e^{(4ix)}+6e^{(2ix)}+1}\left(e^{(ix)}+1\right)}{2\left(e^{(2ix)}-e^{(ix)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(e^(2*I*x) - e^(I*x))*integral(4*sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(
e^(2*I*x) + 1)/(e^(6*I*x) - 2*e^(5*I*x) + 7*e^(4*I*x) - 12*e^(3*I*x) + 7*e^
(2*I*x) - 2*e^(I*x) + 1), x) + sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(e^(I*x) +
 1))/(e^(2*I*x) - e^(I*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos(x)^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(x)^2 - 1), x)
```

maple [B] time = 2.40, size = 75, normalized size = 2.34

$$-\frac{i\sqrt{-\left(1+\cos^2(x)\right)\left(\sin^2(x)\right)}\,\sqrt{1+\cos^2(x)}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\left(2\,\mathrm{EllipticF}\left(i\cos(x), i\right)-\mathrm{EllipticE}\left(i\cos(x), i\right)\right)}{\sqrt{\cos^4(x)-1}\,\sin(x)\sqrt{-1-\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cos(x)^2)^(1/2),x)
```

```
[Out] -I*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*(sin(x)^2)^(1/2)*(2*El
lipticF(I*cos(x),I)-EllipticE(I*cos(x),I))/(cos(x)^4-1)^(1/2)/sin(x)/(-1-co
s(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos(x)^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(x)^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{-{\cos(x)}^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cos(x)^2 - 1)^(1/2),x)
```

```
[Out] int((- cos(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos^2(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(x)**2 - 1), x)
```

3.57 $\int \sqrt{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2
)/(1+b*cos(x)^2/a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (b*Cos[x]^2)/a]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cos^2(x)}\,dx &= \frac{\sqrt{a+b\cos^2(x)}\int \sqrt{1+\frac{b\cos^2(x)}{a}}\,dx}{\sqrt{1+\frac{b\cos^2(x)}{a}}}\\ &= \frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x|-\frac{b}{a}\right)}{\sqrt{1+\frac{b\cos^2(x)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 1.10

$$\frac{\sqrt{2a+b\cos(2x)+b}\,E\left(x\left|\frac{b}{a+b}\right.\right)}{\sqrt{\frac{2a+b\cos(2x)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[2*a + b + b*Cos[2*x]]*EllipticE[x, b/(a + b)])/Sqrt[(2*a + b + b*Cos[
2*x])/(a + b)]
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{b\cos(x)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(x)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(x)^2 + a), x)
```

maple [A] time = 0.97, size = 49, normalized size = 1.17

$$-\frac{a\sqrt{\frac{1}{2} - \frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^2(x)\right)}{a}}\ \operatorname{EllipticE}\left(\cos(x), \sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a + b\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)^(1/2),x)
```

```
[Out] -a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-1/a*b)^(1/2
))/sin(x)/(a+b*cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,{\cos(x)}^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)^(1/2),x)
```

```
[Out] int((a + b*cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos^2{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**2), x)
```

3.58 $\int \left(1+\cos^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=43

$$-\frac{2}{3}F\left(x+\frac{\pi}{2}\middle|-1\right)+2E\left(x+\frac{\pi}{2}\middle|-1\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

```
[Out] -2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)+2/3*(sin(x)^2)^(1/2)/sin(x)*
EllipticF(cos(x),I)+1/3*cos(x)*sin(x)*(1+cos(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3180, 3172, 3177, 3182}

$$-\frac{2}{3}F\left(x+\frac{\pi}{2}\middle|-1\right)+2E\left(x+\frac{\pi}{2}\middle|-1\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(3/2),x]
```

```
[Out] 2*EllipticE[Pi/2 + x, -1] - (2*EllipticF[Pi/2 + x, -1])/3 + (Cos[x]*Sqrt[1
+ Cos[x]^2]*Sin[x])/3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+\cos^2(x)\right)^{3/2}\,dx &= \frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\,\sin(x)+\frac{1}{3}\int \frac{4+6\cos^2(x)}{\sqrt{1+\cos^2(x)}}\,dx\\ &= \frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\,\sin(x)-\frac{2}{3}\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx+2\int \sqrt{1+\cos^2(x)}\,dx\\ &= 2E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{2}{3}F\left(\frac{\pi}{2}+x\middle|-1\right)+\frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\,\sin(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.91

$$\frac{-4F\left(x\middle|\frac{1}{2}\right)+24E\left(x\middle|\frac{1}{2}\right)+\sin(2x)\sqrt{\cos(2x)+3}}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(3/2),x]
[Out] (24*EllipticE[x, 1/2] - 4*EllipticF[x, 1/2] + Sqrt[3 + Cos[2*x]]*Sin[2*x])/
(6*Sqrt[2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos(x)^2+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] integral((cos(x)^2 + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\cos(x)^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((cos(x)^2 + 1)^(3/2), x)
```

maple [B] time = 2.34, size = 101, normalized size = 2.35

$$\frac{\sqrt{\left(1+\cos^2(x)\right)\left(\sin^2(x)\right)}\,\left(-\cos(x)\left(\sin^4(x)\right)+2\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\left(\sin^2(x)\right)+2}\ \text{EllipticF}\left(\cos(x),i\right)-6\sqrt{\frac{1}{2}-\frac{\text{co}}{}}}{3\sqrt{1-\left(\cos^4(x)\right)}\ \sin(x)\sqrt{1+\cos^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)^(3/2),x)
[Out] 1/3*((1+cos(x)^2)*sin(x)^2)^(1/2)*(-cos(x)*sin(x)^4+2*(sin(x)^2)^(1/2)*(-si
n(x)^2+2)^(1/2)*EllipticF(cos(x),I)-6*(sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*
EllipticE(cos(x),I)+2*sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)^
2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\cos(x)^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((cos(x)^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int{\left({\cos\left(x\right)}^2+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)^(3/2),x)
```

```
[Out] int((cos(x)^2 + 1)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cos^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 + 1)**(3/2), x)
```

3.59 $\int \left(-1-\cos^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=89

$$-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}-\frac{2\sqrt{\cos^2(x)+1}\,F\left(x+\frac{\pi}{2}\middle|-1\right)}{3\sqrt{-\cos^2(x)-1}}-\frac{2\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

```
[Out] -1/3*cos(x)*sin(x)*(-1-cos(x)^2)^(1/2)+2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(
cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^2)^(1/2)+2/3*(sin(x)^2)^(1/2)/sin(x
)*EllipticF(cos(x),I)*(1+cos(x)^2)^(1/2)/(-1-cos(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}-\frac{2\sqrt{\cos^2(x)+1}\,F\left(x+\frac{\pi}{2}\middle|-1\right)}{3\sqrt{-\cos^2(x)-1}}-\frac{2\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cos[x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2] - (2*Sq
rt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/(3*Sqrt[-1 - Cos[x]^2]) - (Cos[x]
*Sqrt[-1 - Cos[x]^2]*Sin[x])/3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1-\cos^2(x)\right)^{3/2} dx &= -\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)+\frac{1}{3}\int \frac{4+6\cos^2(x)}{\sqrt{-1-\cos^2(x)}}\,dx\\ &= -\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)-\frac{2}{3}\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx-2\int \sqrt{-1-\cos^2(x)}\,dx\\ &= -\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)-\frac{\left(2\sqrt{-1-\cos^2(x)}\right)\int \sqrt{1+\cos^2(x)}\,dx}{\sqrt{1+\cos^2(x)}}-\frac{\left(2\sqrt{1+\cdots\right.}}{3\cdots}\\ &= -\frac{2\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}-\frac{2\sqrt{1+\cos^2(x)}\,F\left(\frac{\pi}{2}+x\middle|-1\right)}{3\sqrt{-1-\cos^2(x)}}-\frac{1}{3}\cos(x)\sqrt{-1-\cdots}\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.74

$$\frac{6\sin(2x)+\sin(4x)-8\sqrt{\cos(2x)+3}\,F\left(x\middle|\frac{1}{2}\right)+48\sqrt{\cos(2x)+3}\,E\left(x\middle|\frac{1}{2}\right)}{12\sqrt{2}\,\sqrt{-\cos(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cos[x]^2)^(3/2),x]
```

```
[Out] (48*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] - 8*Sqrt[3 + Cos[2*x]]*EllipticF[x
, 1/2] + 6*Sin[2*x] + Sin[4*x])/(12*Sqrt[2]*Sqrt[-3 - Cos[2*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\frac{24\left(e^{(4ix)}-e^{(3ix)}\right)\mathrm{integral}\left(-\frac{4\sqrt{e^{(4ix)}+6e^{(2ix)}+1}\left(5e^{(2ix)}+2e^{(ix)}+5\right)}{3\left(e^{(6ix)}-2e^{(5ix)}+7e^{(4ix)}-12e^{(3ix)}+7e^{(2ix)}-2e^{(ix)}+1\right)},x\right)-\left(e^{(5ix)}-e^{(4ix)}+24e^{(3ix)}+24e^{(2ix}\cdots\right.}{24\left(e^{(4ix)}-e^{(3ix)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*(e^(4*I*x) - e^(3*I*x))*integral(-4/3*sqrt(e^(4*I*x) + 6*e^(2*I*x)
 + 1)*(5*e^(2*I*x) + 2*e^(I*x) + 5)/(e^(6*I*x) - 2*e^(5*I*x) + 7*e^(4*I*x)
- 12*e^(3*I*x) + 7*e^(2*I*x) - 2*e^(I*x) + 1), x) - (e^(5*I*x) - e^(4*I*x)
+ 24*e^(3*I*x) + 24*e^(2*I*x) - e^(I*x) + 1)*sqrt(e^(4*I*x) + 6*e^(2*I*x) +
 1))/(e^(4*I*x) - e^(3*I*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\cos(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x)^2 - 1)^(3/2), x)
```

maple [A] time = 2.62, size = 110, normalized size = 1.24

$$\frac{\sqrt{-\left(1+\cos^2(x)\right)\left(\sin^2(x)\right)}\,\left(-\cos(x)\left(\sin^4(x)\right)+10i\sqrt{-\left(\sin^2(x)\right)+2}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\operatorname{EllipticF}(i\cos(x),i)-6i\sqrt{-}}{3\sqrt{\cos^4(x)-1}\,\sin(x)\sqrt{-1-\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cos(x)^2)^(3/2),x)
```

```
[Out] 1/3*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(-cos(x)*sin(x)^4+10*I*(-sin(x)^2+2)^(1/
2)*(sin(x)^2)^(1/2)*EllipticF(I*cos(x),I)-6*I*(-sin(x)^2+2)^(1/2)*(sin(x)^2
)^(1/2)*EllipticE(I*cos(x),I)+2*sin(x)^2*cos(x))/(cos(x)^4-1)^(1/2)/sin(x)/
(-1-cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cos(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x)^2 - 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(-\cos(x)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int((- cos(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cos^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((-cos(x)**2 - 1)**(3/2), x)
```

3.60 $\int \left(a+b\cos^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,F\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}}+\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{3\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

```
[Out] 1/3*b*cos(x)*sin(x)*(a+b*cos(x)^2)^(1/2)-2/3*(2*a+b)*(sin(x)^2)^(1/2)/sin(x
)*EllipticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2)/(1+b*cos(x)^2/a)^(1/2
)+1/3*a*(a+b)*(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),(-b/a)^(1/2))*(1+b*c
os(x)^2/a)^(1/2)/(a+b*cos(x)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,F\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}}+\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{3\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(3/2),x]
```

```
[Out] (2*(2*a + b)*Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/(3*Sqrt[1 +
(b*Cos[x]^2)/a]) - (a*(a + b)*Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x,
-(b/a)])/(3*Sqrt[a + b*Cos[x]^2]) + (b*Cos[x]*Sqrt[a + b*Cos[x]^2]*Sin[x])/
3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\cos^2(x)\right)^{3/2} dx = \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\sin(x)+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\cos^2(x)}{\sqrt{a+b\cos^2(x)}}dx$$
$$= \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\sin(x)-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\cos^2(x)}}dx+\frac{1}{3}(2(2a+b))\int$$
$$= \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\sin(x)+\frac{\left(2(2a+b)\sqrt{a+b\cos^2(x)}\right)\int\sqrt{1+\frac{b\cos^2(x)}{a}}\,dx}{3\sqrt{1+\frac{b\cos^2(x)}{a}}}-$$
$$= \frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{1+\frac{b\cos^2(x)}{a}}}-\frac{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}\,F\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}}+\frac{1}{3}b\,\mathrm{c}$$

Mathematica [A] time = 0.48, size = 123, normalized size = 1.02

$$\frac{8\left(2a^2+3ab+b^2\right)\sqrt{\frac{2a+b\cos(2x)+b}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right)+\sqrt{2}\,b\sin(2x)(2a+b\cos(2x)+b)-4a(a+b)\sqrt{\frac{2a+b\cos(2x)+b}{a+b}}\,F\left(x\middle|\frac{}{a}\right.}{12\sqrt{2a+b\cos(2x)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(3/2),x]
```

```
[Out] (8*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x,
b/(a + b)] - 4*a*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x,
b/(a + b)] + Sqrt[2]*b*(2*a + b + b*Cos[2*x])*Sin[2*x])/(12*Sqrt[2*a + b +
b*Cos[2*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(x)^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(x)^2 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos(x)^2+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(x)^2 + a)^(3/2), x)
```

maple [A] time = 1.79, size = 192, normalized size = 1.59

$$-\frac{-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{\frac{a+b\left(\cos^2(x)\right)}{a}}\,\mathrm{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{\frac{a+b\left(\cos^2(x)\right)}{a}}\,\mathrm{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{\frac{a+b\left(\cos\right.}{a}}}{}}{\sin(x)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)^(3/2),x)
```

```
[Out] -(-1/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-1/a*b)^
(1/2))*a^2-1/3*a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x)
,(-1/a*b)^(1/2))*b+4/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(
cos(x),(-1/a*b)^(1/2))*a^2+2/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*El
lipticE(cos(x),(-1/a*b)^(1/2))*a*b+1/3*cos(x)^5*b^2+1/3*cos(x)^3*a*b-1/3*co
s(x)^3*b^2-1/3*a*b*cos(x))/sin(x)/(a+b*cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos(x)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(b\cos(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)^(3/2),x)
```

```
[Out] int((a + b*cos(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$F\left(x+\frac{\pi}{2}\middle|-1\right)$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),I)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3182}

$$F\left(x+\frac{\pi}{2}\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] EllipticF[Pi/2 + x, -1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = F\left(\frac{\pi}{2}+x\middle|-1\right)$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.22

$$\frac{F\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] EllipticF[x, 1/2]/Sqrt[2]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\cos(x)^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(cos(x)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(x)^2 + 1), x)
```

maple [B] time = 1.81, size = 41, normalized size = 4.56

$$-\frac{\sqrt{\left(1+\cos^{2}(x)\right)\left(\sin^{2}(x)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\ \mathrm{EllipticF}\left(\cos(x), i\right)}{\sqrt{1-\left(\cos^{4}(x)\right)}\ \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^(1/2),x)
```

```
[Out] -((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(1-cos(x)^4)^(1/2)*Elliptic
F(cos(x),I)/sin(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(x)^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cos(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\sqrt{{\cos(x)}^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int(1/(cos(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos^{2}(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(x)**2 + 1), x)
```

3.62 $\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{\cos^2(x)+1}\,F\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{-\cos^2(x)-1}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),I)*(1+cos(x)^2)^(1/2)/(-1-cos(x)^
2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3183, 3182}

$$\frac{\sqrt{\cos^2(x)+1}\,F\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{-\cos^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] (Sqrt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/Sqrt[-1 - Cos[x]^2]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx &= \frac{\sqrt{1+\cos^2(x)}\,\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx}{\sqrt{-1-\cos^2(x)}}\\ &= \frac{\sqrt{1+\cos^2(x)}\,F\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{-1-\cos^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.03

$$\frac{\sqrt{\cos(2x)+3}\,F\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-\cos(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] (Sqrt[3 + Cos[2*x]]*EllipticF[x, 1/2])/(Sqrt[2]*Sqrt[-3 - Cos[2*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2}{\sqrt{e^{(4ix)} + 6e^{(2ix)} + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] integral(-2/sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x)^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-cos(x)^2 - 1), x)
```

maple [A] time = 2.14, size = 62, normalized size = 1.94

$$\frac{i\sqrt{-\left(1+\cos^2(x)\right)\left(\sin^2(x)\right)}\,\sqrt{1+\cos^2(x)}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\operatorname{EllipticF}(i\cos(x), i)}{\sqrt{\cos^4(x)-1}\,\sin(x)\sqrt{-1-\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cos(x)^2)^(1/2),x)
[Out] I*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(cos(x
)^4-1)^(1/2)*EllipticF(I*cos(x),I)/sin(x)/(-1-cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x)^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-cos(x)^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-{\cos(x)}^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cos(x)^2 - 1)^(1/2),x)
[Out] int(1/(- cos(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos^2(x) - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(-cos(x)**2 - 1), x)
```

3.63 $\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\,F\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),(-b/a)^(1/2))*(1+b*cos(x)^2/a)^(1
/2)/(a+b*cos(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\,F\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -(b/a)])/Sqrt[a + b*Cos[x]^2]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx &= \frac{\sqrt{1+\frac{b\cos^2(x)}{a}}\int \frac{1}{\sqrt{1+\frac{b\cos^2(x)}{a}}}\,dx}{\sqrt{a+b\cos^2(x)}}\\ &= \frac{\sqrt{1+\frac{b\cos^2(x)}{a}}\,F\left(\frac{\pi}{2}+x|-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 46, normalized size = 1.10

$$\frac{\sqrt{\frac{2a+b\cos(2x)+b}{a+b}}\,F\left(x\left|\frac{b}{a+b}\right.\right)}{\sqrt{2a+b\cos(2x)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x, b/(a + b)])/Sqrt[2*a + b
 + b*Cos[2*x]]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cos(x)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(x)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(x)^2 + a), x)
```

maple [A] time = 0.70, size = 48, normalized size = 1.14

$$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^2(x)\right)}{a}}\;\text{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^(1/2),x)
```

```
[Out] -(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-1/a*b)^(1/2))
/sin(x)/(a+b*cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\cos(x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cos^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(x)**2), x)
```

3.64 $\int \frac{1}{\left(1+\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

```
[Out] -1/2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)-1/2*cos(x)*sin(x)/(1+cos(x
)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3184, 21, 3177}

$$\frac{1}{2}E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] EllipticE[Pi/2 + x, -1]/2 - (Cos[x]*Sin[x])/(2*Sqrt[1 + Cos[x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+\cos^2(x)\right)^{3/2}}\,dx &= -\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}-\frac{1}{2}\int \frac{-1-\cos^2(x)}{\sqrt{1+\cos^2(x)}}\,dx\\
&= -\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}+\frac{1}{2}\int \sqrt{1+\cos^2(x)}\,dx\\
&= \frac{1}{2}E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 1.09

$$\frac{E\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}}-\frac{\sin(2x)}{2\sqrt{2}\sqrt{\cos(2x)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] EllipticE[x, 1/2]/Sqrt[2] - Sin[2*x]/(2*Sqrt[2]*Sqrt[3 + Cos[2*x]])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(x)^2+1}}{\cos(x)^4+2\,\cos(x)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(x)^2 + 1)/(cos(x)^4 + 2*cos(x)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos(x)^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x)^2 + 1)^(-3/2), x)
```

maple [B] time = 2.34, size = 70, normalized size = 2.19

$$-\frac{\sqrt{-\left(\sin^4(x)\right)+2\left(\sin^2(x)\right)}\,\left(\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\left(\sin^2(x)\right)+2}\,\operatorname{EllipticE}\left(\cos(x),i\right)+\left(\sin^2(x)\right)\cos(x)\right)}{2\sqrt{1-\left(\cos^4(x)\right)}\,\sin(x)\sqrt{1+\cos^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^(3/2),x)
```

```
[Out] -1/2*(-sin(x)^4+2*sin(x)^2)^(1/2)*((sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*Ell
ipticE(cos(x),I)+sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)^2)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos(x)^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x)^2 + 1)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left({\cos(x)}^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^(3/2),x)
```

```
[Out] int(1/(cos(x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos^2(x)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 + 1)**(-3/2), x)
```

3.65 $\int \frac{1}{\left(-1-\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}+\frac{\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{2\sqrt{\cos^2(x)+1}}$$

```
[Out] 1/2*cos(x)*sin(x)/(-1-cos(x)^2)^(1/2)-1/2*(sin(x)^2)^(1/2)/sin(x)*EllipticE
(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3184, 21, 3178, 3177}

$$\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}+\frac{\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{2\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cos[x]^2)^(-3/2),x]
```

```
[Out] (Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/(2*Sqrt[1 + Cos[x]^2]) + (Cos
[x]*Sin[x])/(2*Sqrt[-1 - Cos[x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1-\cos^2(x)\right)^{3/2}}\,dx &= \frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}-\frac{1}{2}\int \frac{1+\cos^2(x)}{\sqrt{-1-\cos^2(x)}}\,dx\\ &= \frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}+\frac{1}{2}\int \sqrt{-1-\cos^2(x)}\,dx\\ &= \frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}+\frac{\sqrt{-1-\cos^2(x)}\int \sqrt{1+\cos^2(x)}\,dx}{2\sqrt{1+\cos^2(x)}}\\ &= \frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{2\sqrt{1+\cos^2(x)}}+\frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.77

$$\frac{\sin(2x)-2\sqrt{\cos(2x)+3}\,E\left(x\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{-\cos(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cos[x]^2)^(-3/2),x]
[Out] (-2*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] + Sin[2*x])/(2*Sqrt[2]*Sqrt[-3 - C
os[2*x]])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\frac{2\left(e^{(4ix)}+6e^{(2ix)}+1\right)\mathrm{integral}\left(\frac{e^{(2ix)}+3}{2\sqrt{e^{(4ix)}+6e^{(2ix)}+1}},x\right)-\sqrt{e^{(4ix)}+6e^{(2ix)}+1}\left(e^{(3ix)}+3e^{(ix)}\right)}{2\left(e^{(4ix)}+6e^{(2ix)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*(e^(4*I*x) + 6*e^(2*I*x) + 1)*integral(1/2*(e^(2*I*x) + 3)/sqrt(e^(4
*I*x) + 6*e^(2*I*x) + 1), x) - sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(e^(3*I*x)
 + 3*e^(I*x)))/(e^(4*I*x) + 6*e^(2*I*x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\cos(x)^2-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((-cos(x)^2 - 1)^(-3/2), x)
```

maple [A] time = 2.82, size = 101, normalized size = 1.80

$$-\frac{\sqrt{\sin^4(x)-2\left(\sin^2(x)\right)}\,\left(2i\sqrt{-\left(\sin^2(x)\right)+2}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\mathrm{EllipticF}\left(i\cos(x),i\right)-i\sqrt{-\left(\sin^2(x)\right)+2}\,\sqrt{\frac{1}{2}-\frac{\mathrm{c}}{}}}{2\sqrt{\cos^4(x)-1}\,\sin(x)\sqrt{-1-\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cos(x)^2)^(3/2),x)
```

```
[Out] -1/2*(sin(x)^4-2*sin(x)^2)^(1/2)*(2*I*EllipticF(I*cos(x),I)*(sin(x)^2)^(1/2
)*(-sin(x)^2+2)^(1/2)-I*EllipticE(I*cos(x),I)*(sin(x)^2)^(1/2)*(-sin(x)^2+2
)^(1/2)-sin(x)^2*cos(x))/(cos(x)^4-1)^(1/2)/sin(x)/(-1-cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\cos(x)^2 - 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x)^2 - 1)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(-{\cos\left(x\right)}^2-1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int(1/(- cos(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\cos^{2}{\left(x \right)} - 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((-cos(x)**2 - 1)**(-3/2), x)
```

3.66 $\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

```
[Out] -b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^(1/2)-(sin(x)^2)^(1/2)/sin(x)*Ellip
ticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2)/a/(a+b)/(1+b*cos(x)^2/a)^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3184, 21, 3178, 3177}

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/(a*(a + b)*Sqrt[1 + (b*C
os[x]^2)/a]) - (b*Cos[x]*Sin[x])/(a*(a + b)*Sqrt[a + b*Cos[x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx = -\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}} - \frac{\int \frac{-a-b\cos^2(x)}{\sqrt{a+b\cos^2(x)}}\,dx}{a(a+b)}$$
$$= -\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}} + \frac{\int \sqrt{a+b\cos^2(x)}\,dx}{a(a+b)}$$
$$= -\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}} + \frac{\sqrt{a+b\cos^2(x)}\int \sqrt{1+\frac{b\cos^2(x)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}}$$
$$= \frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}} - \frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

Mathematica [A] time = 0.19, size = 75, normalized size = 0.96

$$\frac{2(a+b)\sqrt{\frac{2a+b\cos(2x)+b}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right)-\sqrt{2}\,b\sin(2x)}{2a(a+b)\sqrt{2a+b\cos(2x)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-3/2),x]
[Out] (2*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x, b/(a + b)] - S
qrt[2]*b*Sin[2*x])/(2*a*(a + b)*Sqrt[2*a + b + b*Cos[2*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(x)^2+a}}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(x)^2 + a)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(x)^2 + a)^(-3/2), x)
```

maple [A] time = 1.85, size = 73, normalized size = 0.94

$$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\frac{b\left(\sin^2(x)\right)}{a}+\frac{a+b}{a}}\;a\,\text{EllipticE}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)+b\cos(x)\left(\sin^2(x)\right)}{a\,(a+b)\sin(x)\sqrt{a+b\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^(3/2),x)
```

```
[Out] -((sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*a*EllipticE(cos(x),(-1/a*b
)^(1/2))+b*cos(x)*sin(x)^2)/a/(a+b)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cos(x)^2 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*cos(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cos^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cos(x)**2)**(-3/2), x)
```

3.67 $\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

```
[Out] arcsin(1/2*sin(x)*2^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3186, 216}

$$\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/Sqrt[2]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx, x, \sin(x)\right)\\ &= \sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/Sqrt[2]]
```

fricas [B] time = 0.63, size = 49, normalized size = 5.44

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\cos(x)^2+1}\,\cos(x)^2\sin(x)-\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)+\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt(cos(x)^2 + 1)*cos(x)^2*sin(x) - cos(x)*sin(x))/(cos(x)^4 +
 cos(x)^2 - 1)) + 1/2*arctan(sin(x)/cos(x))
```

giac [A] time = 0.23, size = 8, normalized size = 0.89

$$\arcsin\left(\frac{1}{2}\sqrt{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sqrt(2)*sin(x))
```

maple [B] time = 0.88, size = 33, normalized size = 3.67

$$-\frac{\sqrt{\left(1+\cos^2(x)\right)\left(\sin^2(x)\right)}\arcsin\left(\cos^2(x)\right)}{2\sin(x)\sqrt{1+\cos^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+cos(x)^2)^(1/2),x)
```

```
[Out] -1/2*((1+cos(x)^2)*sin(x)^2)^(1/2)*arcsin(cos(x)^2)/sin(x)/(1+cos(x)^2)^(1/
2)
```

maxima [A] time = 1.61, size = 8, normalized size = 0.89

$$\arcsin\left(\frac{1}{2}\sqrt{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*sqrt(2)*sin(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos(x)}{\sqrt{\cos(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int(cos(x)/(cos(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(cos(x)**2 + 1), x)
```

3.68 $\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x+5)\right)$$

[Out] 1/3*arcsin(1/2*sin(5+3*x))

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x+5)\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[5 + 3*x]/Sqrt[3 + Cos[5 + 3*x]^2],x]

[Out] ArcSin[Sin[5 + 3*x]/2]/3

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(5+3x)\right)\\ &= \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(5+3x)\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x+5)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[5 + 3*x]/Sqrt[3 + Cos[5 + 3*x]^2],x]

[Out] ArcSin[Sin[5 + 3*x]/2]/3

fricas [B] time = 1.39, size = 89, normalized size = 5.93

$$\frac{1}{6}\arctan\left(\frac{\sqrt{\cos(3x+5)^2+3}\left(\cos(3x+5)^2+1\right)\sin(3x+5)-4\cos(3x+5)\sin(3x+5)}{\cos(3x+5)^4+6\cos(3x+5)^2-3}\right)+\frac{1}{6}\arctan\left(\frac{\sin(3}{\cos(3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan((sqrt(cos(3*x + 5)^2 + 3)*(cos(3*x + 5)^2 + 1)*sin(3*x + 5) - 4*
cos(3*x + 5)*sin(3*x + 5))/(cos(3*x + 5)^4 + 6*cos(3*x + 5)^2 - 3)) + 1/6*a
rctan(sin(3*x + 5)/cos(3*x + 5))
```

giac [A] time = 0.42, size = 11, normalized size = 0.73

$$\frac{1}{3} \arcsin\left(\frac{1}{2} \sin\left(3 x + 5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x + 5))
```

maple [B] time = 1.04, size = 57, normalized size = 3.80

$$\frac{\sqrt{\left(3+\cos^{2}\left(5+3x\right)\right)\left(\sin^{2}\left(5+3x\right)\right)}\ \arcsin\left(-1+\frac{\left(\sin^{2}\left(5+3x\right)\right)}{2}\right)}{6\sin\left(5+3x\right)\sqrt{3+\cos^{2}\left(5+3x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x)
```

```
[Out] 1/6*((3+cos(5+3*x)^2)*sin(5+3*x)^2)^(1/2)*arcsin(-1+1/2*sin(5+3*x)^2)/sin(5
+3*x)/(3+cos(5+3*x)^2)^(1/2)
```

maxima [A] time = 0.70, size = 11, normalized size = 0.73

$$\frac{1}{3} \arcsin\left(\frac{1}{2} \sin\left(3 x + 5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x + 5))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cos\left(3 x + 5\right)}{\sqrt{\cos\left(3 x + 5\right)^{2} + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x + 5)/(cos(3*x + 5)^2 + 3)^(1/2),x)
```

```
[Out] int(cos(3*x + 5)/(cos(3*x + 5)^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(3x + 5\right)}{\sqrt{\cos^{2}\left(3x + 5\right) + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)**2)**(1/2),x)
```

```
[Out] Integral(cos(3*x + 5)/sqrt(cos(3*x + 5)**2 + 3), x)
```

3.69 $\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

```
[Out] arcsinh(1/3*sin(x)*3^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3186, 215}

$$\sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[4 - Cos[x]^2],x]
[Out] ArcSinh[Sin[x]/Sqrt[3]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sin(x)\right)\\ &= \sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[4 - Cos[x]^2],x]
[Out] ArcSinh[Sin[x]/Sqrt[3]]
```

fricas [B] time = 1.21, size = 39, normalized size = 4.33

$$\frac{1}{4}\log\left(8\cos(x)^4 - 4\left(2\cos(x)^2 - 5\right)\sqrt{-\cos(x)^2 + 4}\sin(x) - 40\cos(x)^2 + 41\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 5)*sqrt(-cos(x)^2 + 4)*sin(x) - 40*cos
(x)^2 + 41)
```

giac [A] time = 1.42, size = 16, normalized size = 1.78

$$-\log\left(\sqrt{\sin(x)^2+3}-\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(sin(x)^2 + 3) - sin(x))
```

maple [B] time = 2.15, size = 53, normalized size = 5.89

$$-\frac{\sqrt{-\left(-4+\cos^2(x)\right)\left(\sin^2(x)\right)}\,\ln\left(-\left(\sin^2(x)\right)+\sqrt{\sin^4(x)+3\left(\sin^2(x)\right)}-\frac{3}{2}\right)}{2\sin(x)\sqrt{4-\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4-cos(x)^2)^(1/2),x)
```

```
[Out] -1/2*(-(-4+cos(x)^2)*sin(x)^2)^(1/2)*ln(-sin(x)^2+(sin(x)^4+3*sin(x)^2)^(1/
2)-3/2)/sin(x)/(4-cos(x)^2)^(1/2)
```

maxima [A] time = 1.74, size = 8, normalized size = 0.89

$$\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(3)*sin(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos(x)}{\sqrt{4-\cos(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4 - cos(x)^2)^(1/2),x)
```

```
[Out] int(cos(x)/(4 - cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{-(\cos(x)-2)(\cos(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(-(cos(x) - 2)*(cos(x) + 2)), x)
```

3.70 $\int \frac{1}{a+b\cos^4(x)}\,dx$

Optimal. Leaf size=487

$$-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left((a+b)^{3/4}\cot^2(x)-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cot(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\mathrm{l}}{}$$

```
[Out] -1/8*ln((a+b)^(3/4)*cot(x)^2+(a+b)^(1/4)*a^(1/2)-a^(1/4)*cot(x)*2^(1/2)*(a+
b-a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(
1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+1/8*ln((a+b)^(3/4)*cot(x)^2+(a+b)^(1/4
)*a^(1/2)+a^(1/4)*cot(x)*2^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)-
(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+1/
4*arctan((-(a+b)^(3/4)*cot(x)*2^(1/2)+a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/
2))/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/
(a+b)^(1/4)*2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)-1/4*arctan(((a+b)^(3/4)
*cot(x)*2^(1/2)+a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2))/a^(1/4)/(a+b+a^(1/
2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a
+b+a^(1/2)*(a+b)^(1/2))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left((a+b)^{3/4}\cot^2(x)-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cot(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^4)^(-1),x]
```

```
[Out] ((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
- Sqrt[2]*(a + b)^(3/4)*Cot[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])
])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((
Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]] +
Sqrt[2]*(a + b)^(3/4)*Cot[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])])
/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((Sq
rt[a] - Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b
 - Sqrt[a]*Sqrt[a + b]]*Cot[x] + (a + b)^(3/4)*Cot[x]^2])/(4*Sqrt[2]*a^(3/4
)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) + ((Sqrt[a] - Sqrt[a + b
])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a
+ b]]*Cot[x] + (a + b)^(3/4)*Cot[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sq
rt[a + b - Sqrt[a]*Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a+b\cos^4(x)}\,dx = -\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a+b)x^4}\,dx, x, \cot(x)\right)$$

$$= -\frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)x}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \cot(x)\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} - \frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{\frac{\sqrt{a}}{\sqrt{a+b}}+\cdots}\right.}{2\sqrt{2}\,a^{3/4}\sqrt{\cdots}}$$

$$= -\frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \cot(x)\right)}{4(a+b)} - \frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,\cot(x)+(a+b)^{3/4}\cdots\right.}{4\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$

$$= \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\cot(x)\right)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} - \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\cdots}{\cdots}\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a\cdots}}$$

Mathematica [C] time = 0.24, size = 121, normalized size = 0.25

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + I*Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + I*
Sqrt[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + I*Sqrt[a]*Sqrt[b]]]/
(2*Sqrt[a]*Sqrt[-a + I*Sqrt[a]*Sqrt[b]])
```

fricas [B] time = 0.86, size = 809, normalized size = 1.66

$$-\frac{1}{8}\sqrt{-\frac{\left(a^2+ab\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\left(b\cos(x)^2+2\left(ab\cos(x)\sin(x)+\left(a^4+a^3b\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)
)*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^
4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b +
a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt
(-b/(a^5 + 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) +
(a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2
 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3 + a^2*b
 - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) + 1/8*sqrt
(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(-b*c
os(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3
*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))
- 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 +
 2*a^4*b + a^3*b^2))) - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*
b^2)) - 1)/(a^2 + a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) - (a^4 + a^3
*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqr
t(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 +
 a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))
```

giac [A] time = 1.03, size = 307, normalized size = 0.63

$$\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^2+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab-3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\text{arc}\right.}{2\left(3a^5+7a^4b+4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b - 3*sq
rt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*b)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)*a
+ 16*a^2))/a)))*abs(a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sq
```

```
rt(-a*b)*a)*a^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b + 3*sqrt(a^2 - sqrt(-a*b)*
a)*sqrt(-a*b)*a + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*(pi*floor(x/pi +
 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/a)))*abs(
a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2)
```

maple [B] time = 0.23, size = 3350, normalized size = 6.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^4),x)
```

```
[Out] -1/8/(a+b)^(3/2)*ln(a^(1/2)*tan(x)^2-tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(
a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/(a+b)^(3/2)*ln(a^(1/2)*tan(x)
^2+tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a
)^(1/2)+a/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*a
rctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))+b/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a
+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1
/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))+a/(a+b)^(3/2)/(4*
a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)+(
2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)
^(1/2))+b/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*a
rctan((2*a^(1/2)*tan(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))-1/8/a^(1/2)/(a+b)*ln(a^(1/2)*tan(x)^2+tan(
x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
+1/8/a^(1/2)/(a+b)*ln(a^(1/2)*tan(x)^2-tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)
+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8*a/b/(a+b)^(3/2)*ln(a^(1/2)*
tan(x)^2+tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/
2)-2*a)^(1/2)+1/8/b/(a+b)^(3/2)*ln(a^(1/2)*tan(x)^2+tan(x)*(2*((a+b)*a)^(1/
2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/
8/a/(a+b)^(3/2)*ln(a^(1/2)*tan(x)^2+tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a
+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/a^(1/2)/b/(a+b
)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan
(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)
+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a
^2+a*b)^(1/2)+1/4/a^(1/2)/b/(a+b)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+
2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/
2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*
(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/a/(a+b)^(3/2)/(4*a^(1/2)*
(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)
*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*
(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)
+1/4/a^(3/2)/(a+b)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arct
an((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-
2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1
/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(1/2)/b/(a+b)/(4*a^(1/2)*(a+b)^(1/2)-2
*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a
)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^
(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(3/2)/(a+b)/(4*a^(1/2)
*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)+(2*((a+b
)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))
*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2
)+1/4*a^(1/2)/b/(a+b)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*a
rctan((2*a^(1/2)*tan(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)
^(1/2)-2*a)^(1/2)-1/4*a/b/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1
/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a
^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1
/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/b/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-
```

```
2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*
a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)
^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4*a/b/(a+
b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1
/2)*tan(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a
)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(
1/2)-1/4/b/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*
arctan((2*a^(1/2)*tan(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1
/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b
)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/a/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)
-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)+(2*((a+b)*a)^(1/2)-2
*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a
)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8/a^(3/2
)/(a+b)*ln(a^(1/2)*tan(x)^2+tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2
))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8*a^(1/2)/b/(a+b)*ln(a^(
1/2)*tan(x)^2+tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b
)^(1/2)-2*a)^(1/2)-1/4/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)
+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1
/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)
*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8*a^(1/2)/b/(a+b)*ln(a^(1/2)*tan(x)^2-tan(
x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
+1/4/a^(1/2)/(a+b)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arct
an((2*a^(1/2)*tan(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-
2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1
/2)-2*a)^(1/2)-1/8*a/b/(a+b)^(3/2)*ln(a^(1/2)*tan(x)^2-tan(x)*(2*((a+b)*a)^
(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/b/(a+b)^(3/
2)*ln(a^(1/2)*tan(x)^2-tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2
*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8/a/(a+b)^(3/2)*ln(a^(1/2)*ta
n(x)^2-tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)
-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/(a+b)^(3/2)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)
*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2)
)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2
*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^(3/2)/(a+b)*ln(a^(1/2)*tan(x)
^2-tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a
)^(1/2)*(a^2+a*b)^(1/2)+1/8/a^(1/2)/b/(a+b)*ln(a^(1/2)*tan(x)^2-tan(x)*(2*(
(a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a
*b)^(1/2)-1/8/a^(1/2)/b/(a+b)*ln(a^(1/2)*tan(x)^2+tan(x)*(2*((a+b)*a)^(1/2)
-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/
a^(1/2)/(a+b)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2
*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a
+b)*a)^(1/2)+2*a)^(1/2))*(2*((a+b)*a)^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2
*a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cos(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^4 + a), x)
```

mupad [B] time = 2.66, size = 926, normalized size = 1.90

$$-2\,\mathrm{atanh}\left(\frac{8\,a^6\,b\tan(x)\sqrt{-\frac{a^2}{16\left(a^4+b\,a^3\right)}-\frac{\sqrt{-a^3\,b}}{16\left(a^4+b\,a^3\right)}}}{\frac{2\,a^9\,b}{a^4+b\,a^3}-2\,a^4\,b^2-2\,a^5\,b+\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}}-\frac{8\,a^2\,b\tan(x)\sqrt{-\frac{a^2}{16\left(a^4+b\,a^3\right)}-\frac{\sqrt{-a^3\,b}}{16\left(a^4+b\,a^3\right)}}}{\frac{2\,a^5\,b}{a^4+b\,a^3}-2\,a\,b+\frac{2\,a^3\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^4),x)
```

```
[Out] - 2*atanh((8*a^6*b*tan(x)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(a
^3*b + a^4)))^(1/2))/((2*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*a^
8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*
(-a^3*b)^(1/2))/(a^3*b + a^4)) - (8*a^2*b*tan(x)*(- a^2/(16*(a^3*b + a^4))
- (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/2))/((2*a^5*b)/(a^3*b + a^4) - 2*a*
b + (2*a^3*b*(-a^3*b)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b)^(1/2
)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/2))/((2
*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*a^8*b^2)/(a^3*b + a^4) + (
2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))/(a^3*b +
 a^4)))*(-(a^2 + (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2) - 2*atanh((8*a^2
*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b + a^4)))^(1/2
))/(2*a*b - (2*a^5*b)/(a^3*b + a^4) + (2*a^3*b*(-a^3*b)^(1/2))/(a^3*b + a^4
)) - (8*a^6*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b +
a^4)))^(1/2))/(2*a^5*b + 2*a^4*b^2 - (2*a^9*b)/(a^3*b + a^4) - (2*a^8*b^2)/
(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b
)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b)^(1/2)*((-a^3*b)^(1/2)/(1
6*(a^3*b + a^4)) - a^2/(16*(a^3*b + a^4)))^(1/2))/(2*a^5*b + 2*a^4*b^2 - (2
*a^9*b)/(a^3*b + a^4) - (2*a^8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2)
)/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))/(a^3*b + a^4)))*(-(a^2 - (-a^3
*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**4),x)
```

```
[Out] Timed out
```

3.71 $\int \frac{1}{a-b\cos^4(x)}\,dx$

Optimal. Leaf size=101

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] -1/2*arctan(cot(x)*(a^(1/2)-b^(1/2))^(1/2)/a^(1/4))/a^(3/4)/(a^(1/2)-b^(1/2
))^(1/2)-1/2*arctan(cot(x)*(a^(1/2)+b^(1/2))^(1/2)/a^(1/4))/a^(3/4)/(a^(1/2
)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3209, 1166, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Cot[x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt[a] -
 Sqrt[b]]) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Cot[x])/a^(1/4)]/(2*a^(3/4)*Sq
rt[Sqrt[a] + Sqrt[b]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a-b\cos^4(x)}\,dx = -\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \cot(x)\right)\right) - \frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int\right.$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Mathematica [A] time = 0.20, size = 109, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cos[x]^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + Sqrt
[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*Sqr
t[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]])
```

fricas [B] time = 0.71, size = 817, normalized size = 8.09

$$-\frac{1}{8}\sqrt{-\frac{\left(a^2-ab\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}+1}{a^2-ab}}\log\left(b\cos(x)^2+2\left(ab\cos(x)\sin(x)-\left(a^4-a^3b\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b))
*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*
b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3
*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/
(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b
+ a^3*b^2)) + 1)/(a^2 - a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) - (a^4
- a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b
)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a
^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqrt(((a^2 -
 a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(-b*cos(x)^2 +
 2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos
(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 -
 a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a
^3*b^2))) - 1/8*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a
^2 - a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(a
^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a
^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^
2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))
```

giac [B] time = 1.13, size = 299, normalized size = 2.96

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2-4\sqrt{a^2+\sqrt{ab}\,a}\,ab-3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a+4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\quad}{\sqrt{\frac{4a}{}}}\right.\right.}{2\left(3a^5-7a^4b+4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2 - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b - 3*sqrt
(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b)*(p
i*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a - b)*a + 16*a
^2))/a)))*abs(a)/(3*a^5 - 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt(a*b
)*a)*a^2 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b + 3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a
*b)*a - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*(pi*floor(x/pi + 1/2) + arct
an(2*tan(x)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/a)))*abs(a)/(3*a^5 -
7*a^4*b + 4*a^3*b^2)
```

maple [A] time = 0.09, size = 64, normalized size = 0.63

$$-\frac{\operatorname{arctanh}\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2\sqrt{\left(\sqrt{ab}+a\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^4),x)
```

```
[Out] -1/2/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*tan(x)/(((a*b)^(1/2)-a)*a)^(1/2))+
1/2/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*tan(x)/(((a*b)^(1/2)+a)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{b\cos(x)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^4 - a), x)
```

mupad [B] time = 2.60, size = 938, normalized size = 9.29

$$2\,\operatorname{atanh}\left(\frac{8a^6b\tan(x)\sqrt{\frac{a^2}{16\left(a^3b-a^4\right)}+\frac{\sqrt{a^3b}}{16\left(a^3b-a^4\right)}}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}-\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}+\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}-\frac{8a^2b\tan(x)\sqrt{\frac{a^2}{16\left(a^3b-a^4\right)}+\frac{\sqrt{a^3b}}{16\left(a^3b-a^4\right)}}}{2ab+\frac{2a^5b}{a^3b-a^4}+\frac{2a^3b\sqrt{a^3b}}{a^3b-a^4}}+\frac{\quad}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^4),x)
```

```
[Out] 2*atanh((8*a^6*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^3*b)^(1/2)/(16*(a^3*b
- a^4)))^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b
)/(a^3*b - a^4) - (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^3*b
)^(1/2))/(a^3*b - a^4)) - (8*a^2*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^3*b)
```

```
^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b + (2*a^5*b)/(a^3*b - a^4) + (2*a^3
*b*(a^3*b)^(1/2))/(a^3*b - a^4)) + (8*a^4*b*tan(x)*(a^2/(16*(a^3*b - a^4))
+ (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b - 2*a^4*b
^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) - (2*a^6*b^2*(a^3*
b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4)))*((a^2 + (
a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) - 2*atanh((8*a^2*b*tan(x)*(a^2/(16*
(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b + (2*a^5*b
)/(a^3*b - a^4) - (2*a^3*b*(a^3*b)^(1/2))/(a^3*b - a^4)) - (8*a^6*b*tan(x)*
(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a^5*b
 - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) + (2*a^6
*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) - (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4))
+ (8*a^4*b*tan(x)*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)
))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) +
(2*a^9*b)/(a^3*b - a^4) + (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) - (2*a^7*
b*(a^3*b)^(1/2))/(a^3*b - a^4)))*((a^2 - (a^3*b)^(1/2))/(16*(a^3*b - a^4)))
^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**4),x)
```

```
[Out] Timed out
```

3.72 $\int \frac{1}{1+\cos^4(x)}\,dx$

Optimal. Leaf size=292

$$\frac{x}{2\sqrt{\sqrt{2}-1}}+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(2\cot^2(x)-2\sqrt{\sqrt{2}-1}\cot(x)+\sqrt{2}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}\cot^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\right.$$

```
[Out] 1/2*x/(2^(1/2)-1)^(1/2)+1/4*arctan((cos(x)*sin(x)*(-2+2^(1/2))+(-1+2*sin(x)
^2)*(2^(1/2)-1)^(1/2))/(2+sin(x)^2*(-2+2^(1/2))-2*cos(x)*sin(x)*(2^(1/2)-1)
^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)+1/4*arctan((cos(x)*sin(x)*(-2+
2^(1/2))+(1-2*sin(x)^2)*(2^(1/2)-1)^(1/2))/(2+sin(x)^2*(-2+2^(1/2))+2*cos(x
)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)+1/8*ln(2*c
ot(x)^2+2^(1/2)-2*cot(x)*(2^(1/2)-1)^(1/2))*(2^(1/2)-1)^(1/2)-1/8*ln(1+cot(
x)^2*2^(1/2)+cot(x)*(-2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3209, 1169, 634, 618, 204, 628}

$$\frac{x}{2\sqrt{\sqrt{2}-1}}+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(2\cot^2(x)-2\sqrt{\sqrt{2}-1}\cot(x)+\sqrt{2}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}\cot^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[-1 + Sqrt[2]]) + ArcTan[((-2 + Sqrt[2])*Cos[x]*Sin[x] + Sqrt[-1 +
 Sqrt[2]]*(1 - 2*Sin[x]^2))/(2 + Sqrt[1 + Sqrt[2]] + 2*Sqrt[-1 + Sqrt[2]]*C
os[x]*Sin[x] + (-2 + Sqrt[2])*Sin[x]^2)]/(4*Sqrt[-1 + Sqrt[2]]) + ArcTan[((
-2 + Sqrt[2])*Cos[x]*Sin[x] + Sqrt[-1 + Sqrt[2]]*(-1 + 2*Sin[x]^2))/(2 + Sq
rt[1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x] + (-2 + Sqrt[2])*Sin[x
]^2)]/(4*Sqrt[-1 + Sqrt[2]]) + (Sqrt[-1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[-1
+ Sqrt[2]]*Cot[x] + 2*Cot[x]^2])/8 - (Sqrt[-1 + Sqrt[2]]*Log[1 + Sqrt[2*(-1
 + Sqrt[2])]*Cot[x] + Sqrt[2]*Cot[x]^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{1+\cos^4(x)}\,dx = -\text{Subst}\left(\int \frac{1+x^2}{1+2x^2+2x^4}\,dx, x, \cot(x)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}-\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \cot(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}+\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \cot(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}$$

$$= \frac{1}{8}\sqrt{-1+\sqrt{2}}\ \text{Subst}\left(\int \frac{-\sqrt{-1+\sqrt{2}}+2x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \cot(x)\right) - \frac{1}{8}\sqrt{-1+\sqrt{2}}\ \text{Subst}\left(\int \ldots\right.$$

$$= \frac{1}{8}\sqrt{-1+\sqrt{2}}\ \log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\ \cot(x)+2\cot^2(x)\right) - \frac{1}{8}\sqrt{-1+\sqrt{2}}\ \log\left(1+\sqrt{2\left(-\ldots\right.}\right.$$

$$= \frac{1}{2}\sqrt{1+\sqrt{2}}\,x - \frac{1}{4}\sqrt{1+\sqrt{2}}\ \tan^{-1}\left(\frac{\left(2-\sqrt{2}\right)\cos(x)\sin(x)-\sqrt{-1+\sqrt{2}}\ \left(1-2\sin^2(\ldots\right.}{2+\sqrt{1+\sqrt{2}}\ +2\sqrt{-1+\sqrt{2}}\ \cos(x)\sin(x)-\left(2-\sqrt{2}\ldots\right.}\right.$$

Mathematica [C] time = 0.08, size = 45, normalized size = 0.15

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{2\sqrt{1-i}}+\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^4)^(-1),x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTan[Tan[x]/Sqrt[1 + I]]/(2*
Sqrt[1 + I])
```

fricas [B] time = 42.42, size = 3830, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/32*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)
^4 + 4*(sqrt(2) - 1)*cos(x)^2 + (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)
*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 1) + 1/32*2^(1/4)*sqrt(
2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)^4 + 4*(sqrt(2) - 1
)*cos(x)^2 - (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(
x))*sqrt(2*sqrt(2) + 4)*sin(x) + 1) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*arct
an(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2
)*(19*sqrt(2) + 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) +
19) + 2*sqrt(2) - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(
2) + 39)*cos(x)^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^
8 + 4*(sqrt(2)*(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6
*sqrt(2) + 1) - 2*sqrt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2
*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4
)*(4*sqrt(2) + 5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*
sqrt(2) + 58))*cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(
2) - 32))*cos(x)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) -
 2))*cos(x)^7 + 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*co
s(x)^5 - (2*2^(3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^
(3/4)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 + (16*(sqrt(2)*(5*sqr
t(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt
(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)
^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)
*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2
) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8
*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(
3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)
*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*s
qrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2
) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) -
 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*s
qrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)
*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos
(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*
cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*
cos(x)^2 - 1)) + 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*
(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 22)
- 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) - 37
)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^10 -
 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(49*s
qrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) - 2*sq
rt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2)
+ 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) + 5))*c
os(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*cos(x
)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x)^9 -
 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 + 2*(2
^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(3/4)*
(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*sqrt(2
*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2
) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 +
 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7
*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) -
 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*
cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2)
 - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11
) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) -
 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/
```

```
4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(
4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2)
- 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) -
 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 + 4*(2^(1/
4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2)
+ 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(
x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) - 1/1
6*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*
sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 22) - 8*sqrt(2) - 52)*co
s(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) - 37)*cos(x)^12 + 16*(2*
sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^10 - 8*(sqrt(2)*(41*sqrt
(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(49*sqrt(2) + 6) - 32*sqr
t(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) - 2*sqrt(2) - 5)*cos(x)^4
- 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^15 - 8*
(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) + 5))*cos(x)^13 - 4*(2*2^(3
/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*cos(x)^11 + 4*(6*2^(3/4)*
(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x)^9 - 2*(2^(3/4)*(28*sqrt
(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 + 2*(2^(3/4)*(9*sqrt(2) -
8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(3/4)*(sqrt(2) - 1) - 2^(1
/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x)
 + 4*cos(x)^2 + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 5
6*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt
(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*
sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*co
s(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2
)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(
5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqr
t(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2)
 - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34)
)*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(
x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(
1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(
2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 - 4*(2^(1/4)*(3*sqrt(2) - 4)*c
os(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4))/(1
12*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^8
 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) + 1/16*2^(1/4)*sqrt(2*sqr
t(2) + 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^1
6 - 16*(sqrt(2)*(19*sqrt(2) + 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)
*(8*sqrt(2) + 19) + 2*sqrt(2) - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) -
13) - 22*sqrt(2) + 39)*cos(x)^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2
) - 2)*cos(x)^8 + 4*(sqrt(2)*(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 -
 8*(sqrt(2)*(6*sqrt(2) + 1) - 2*sqrt(2) - 5)*cos(x)^4 - 2*(8*(2^(3/4)*(2*sq
rt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) +
 2) - 4*2^(1/4)*(4*sqrt(2) + 5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10)
+ 2^(1/4)*(19*sqrt(2) + 58))*cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(
1/4)*(19*sqrt(2) - 32))*cos(x)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)
*(15*sqrt(2) - 2))*cos(x)^7 + 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sq
rt(2) + 2))*cos(x)^5 - (2*2^(3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))
*cos(x)^3 - 2^(3/4)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 - (16*(
sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2)
 - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2
) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^
8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2
)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)
*cos(x)^2 - (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x
)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11
+ 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*
(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(
3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*
```

```
sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*co
s(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16
*(sqrt(2) - 1)*cos(x)^2 - 4*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sq
rt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*co
s(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*
cos(x)^4 + 16*cos(x)^2 - 1))
```

giac [A] time = 0.67, size = 170, normalized size = 0.58

$$\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+\right.}{2\sqrt{\sqrt{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2)
+ 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x/pi + 1/
2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sqrt(
2) + 2)))*sqrt(sqrt(2) + 1) - 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^2 + 2^(1/4)*
sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^2 -
 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2))
```

maple [A] time = 0.22, size = 227, normalized size = 0.78

$$\frac{\sqrt{2}\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{16}+\frac{\arctan\left(\frac{2\tan(x)-\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right)\sqrt{2}}{4\sqrt{2\sqrt{2}+2}}+\frac{\arctan\left(\frac{2\tan(x)-\sqrt{-2+}}{\sqrt{2\sqrt{2}+2}}\right.}{2\sqrt{2\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^4),x)
```

```
[Out] 1/16*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(tan(x)^2-tan(x)*(-2+2*2^(1/2))^(1/2)+2
^(1/2))+1/4/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)-(-2+2*2^(1/2))^(1/2))/(2*2
^(1/2)+2)^(1/2))*2^(1/2)+1/2/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)-(-2+2*2^(
1/2))^(1/2))/(2*2^(1/2)+2)^(1/2))-1/16*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(tan(
x)^2+tan(x)*(-2+2*2^(1/2))^(1/2)+2^(1/2))+1/4/(2*2^(1/2)+2)^(1/2)*arctan((2
*tan(x)+(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2))*2^(1/2)+1/2/(2*2^(1/2)+2
)^(1/2)*arctan((2*tan(x)+(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(x)^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(cos(x)^4 + 1), x)
```

mupad [B] time = 2.73, size = 214, normalized size = 0.73

$$\mathrm{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}+\frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)-\mathrm{at}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4 + 1),x)
```

```
[Out] atanh((4*2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64
)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) + (4*2^(1/2)*tan(x)*(2^(1/2)/64 -
1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1)
)*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2)) - atanh((4*
2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(
- 2^(1/2)/64 - 1/64)^(1/2) + 1) - (4*2^(1/2)*tan(x)*(2^(1/2)/64 - 1/64)^(1/
2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1))*(2*(- 2
^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**4),x)
```

```
[Out] Timed out
```

3.73 $\int \frac{1}{1-\cos^4(x)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2\sqrt{2}}-\frac{\cot(x)}{2}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{2\sqrt{2}}$$

```
[Out] -1/2*cot(x)+1/4*x*2^(1/2)-1/4*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3209, 388, 203}

$$\frac{x}{2\sqrt{2}}-\frac{\cot(x)}{2}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[2]) - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/(2*Sqrt[2]
) - Cot[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-\cos^4(x)}\,dx &= -\text{Subst}\left(\int \frac{1+x^2}{1+2x^2}\,dx,x,\cot(x)\right)\\
&= -\frac{\cot(x)}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\
&= \frac{x}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{2\sqrt{2}}-\frac{\cot(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 24, normalized size = 0.53

$$\frac{1}{4}\left(\sqrt{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)-2\cot(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^4)^(-1),x]
[Out] (Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/4
```

fricas [A] time = 0.68, size = 43, normalized size = 0.96

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)\sin(x)+4\cos(x)}{8\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^4),x, algorithm="fricas")
[Out] -1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))*si
n(x) + 4*cos(x))/sin(x)
```

giac [A] time = 0.56, size = 53, normalized size = 1.18

$$\frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^4),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - 1/2/tan(x)
```

maple [A] time = 0.08, size = 21, normalized size = 0.47

$$\frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^4),x)
[Out] 1/4*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)-1/2/tan(x)
```

maxima [A] time = 1.64, size = 20, normalized size = 0.44

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^4),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/2/tan(x)
```

mupad [B] time = 2.16, size = 20, normalized size = 0.44

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^4 - 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*tan(x))/2))/4 - 1/(2*tan(x))
```

sympy [A] time = 1.98, size = 78, normalized size = 1.73

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}+\frac{\tan\left(\frac{x}{2}\right)}{4}-\frac{1}{4\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**4),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 + sqrt(2
)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4 + tan(x/2)/4 -
 1/(4*tan(x/2))
```

3.74 $\int \frac{1}{a+b\cos^5(x)}\,dx$

Optimal. Leaf size=494

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+(-1\cdots}}$$

```
[Out] 2/5*arctan((a^(1/5)-b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+b^(1/5))^(1/2))/a^(4
/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1
)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2))/a^(4/
5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2)+2/
5*arctan((a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(2/5)*
b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(2
/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)*tan(1/2*x)
/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(
1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(4/5)*b^(1
/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-
(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3213, 2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\,\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+(-1\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a^(1/5) - b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + b^(1/5)]])/(5*a
^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt[a
^(1/5) + (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]])
/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5)*b^
(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1
/5) + (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*S
qrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^(3/5)*b
^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1
/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2*ArcTan[(
Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(4/5)*b^(1
/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(4
/5)*b^(1/5)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \int\left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{\ldots}\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{\ldots}}}{5a^{4/5}}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}+\left(-\sqrt[5]{a}+\sqrt[5]{b}\right)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}+\left(-\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}\right)x^{\ldots}}\right.}{5a^{4/5}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\,\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\tan^{-1}}{5a^{4/5}\sqrt{\sqrt[5]{a}-(\ldots}}$$

Mathematica [C] time = 0.25, size = 130, normalized size = 0.26

$$\frac{8}{5}\mathrm{RootSum}\left[\#1^{10}b+5\#1^8b+10\#1^6b+32\#1^5a+10\#1^4b+5\#1^2b+b\&,\frac{2\#1^3\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\#1^3\log\left(\#1^2-\ldots\right.}{\#1^8b+4\#1^6b+6\#1^4b+16\#1^3a+\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x]^5)^(-1),x]
```

```
[Out] (8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b*
#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 + #1
^2]*#1^3)/(b + 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])/5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cos(x)^5+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*cos(x)^5 + a), x)
```

maple [C] time = 0.10, size = 150, normalized size = 0.30

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{10}+(5a+5b)\_Z^{8}+(10a-10b)\_Z^{6}+(10a+10b)\_Z^{4}+(5a-5b)\_Z^{2}+a+b\right)} \frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a-\_R^{9}b+4\_R^{7}a+4\_R^{7}b+6\_R^{5}a-6\_R^{5}b+4\_R^{3}a+4\_R^{3}b+\_Ra-\_Rb}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^5),x)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a-_R^9*b+4*_R^7*a+4*_R^7*b+6*_R
^5*a-6*_R^5*b+4*_R^3*a+4*_R^3*b+_R*a-_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a-b
)*_Z^10+(5*a+5*b)*_Z^8+(10*a-10*b)*_Z^6+(10*a+10*b)*_Z^4+(5*a-5*b)*_Z^2+a+b
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cos(x)^{5}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^5 + a), x)
```

mupad [B] time = 8.82, size = 1520, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^5),x)
```

```
[Out] symsum(log(-(10995116277760*b^7*(a - b)*(7*cot(x/2) - 56*root(9765625*a^8*b
^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d
^4 - 125*a^2*d^2 - 1, d, k)*a + root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^
10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d,
k)*b - 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 - 225000*ro
ot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*
d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 - 3875000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 - 25000000*root(9765625*a^8*b^2*d^10 -
 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*
a^2*d^2 - 1, d, k)^9*a^9 + 735*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1
0 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k
)^2*a^2*cot(x/2) + 28875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 19
53125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^
4*cot(x/2) + 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125
*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^6*cot
(x/2) + 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^8*cot(x/2
) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b + 100000*root
(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^
6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b + 4000000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b + 50000000*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^9*a^8*b - 125000*root(9765625*a^8*b^2*d^10 - 9765625*a
```

```
^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 -
1, d, k)^7*a^5*b^2 - 25000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^9*a^7*b^2 - 35*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a*b*cot(x/2
) - 7000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^3*b*cot(x/2) - 3
50000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156
250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b*cot(x/2) - 5000
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 15625
0*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b*cot(x/2) + 3125*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*cot(x/2) + 1718750*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*cot(x/2)))/cot(x/2))
*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cos^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**5),x)
```

```
[Out] Integral(1/(a + b*cos(x)**5), x)
```

3.75 $\int \frac{1}{a+b\cos^6(x)}\,dx$

Optimal. Leaf size=171

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] -1/3*arctan(cot(x)*(a^(1/3)+b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/3
))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/a^(1/6))/a^(5
/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)+(-1)^(2/3
)*b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^6)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
 b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3
*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) + (-1)^
(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3
)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}dx, x, \cot(x)\right)}{3a} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}dx, x, \cot(x)\right)}{3a} - \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.23, size = 146, normalized size = 0.85

$$\frac{8}{3}\text{RootSum}\left[\#1^6b+6\#1^5b+15\#1^4b+64\#1^3a+20\#1^3b+15\#1^2b+6\#1b+b\&, \frac{2\#1^2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^2\text{lo}}{\#1^5b+5\#1^4b+10\#1^3b+32\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^6)^(-1),x]
```

```
[Out] (8*RootSum[b + 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*b
*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*C
os[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 + 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 +
5*b*#1^4 + b*#1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^6),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.46, size = 60, normalized size = 0.35

$$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+3a\_Z^2+a+b\right)} \frac{\left(\_R^4+2\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^5+2\_R^3+\_R}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^6),x)
```

```
[Out] 1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+3
*_Z^4*a+3*_Z^2*a+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cos(x)^6 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^6 + a), x)
```

mupad [B] time = 3.08, size = 184, normalized size = 1.08

$$\sum_{k=1}^{6} \ln\Big(\mathrm{root}\big(46656\,a^5\,b\,d^6 + 46656\,a^6\,d^6 + 3888\,a^4\,d^4 + 108\,a^2\,d^2 + 1, d, k\big)^2\,a^3\,b^3\,\Big(\mathrm{root}\big(46656\,a^5\,b\,d^6 + 4665$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^6),x)
```

```
[Out] symsum(log(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2
*d^2 + 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a
^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 + 46656*
a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(46656*a
^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cos^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**6),x)
```

```
[Out] Integral(1/(a + b*cos(x)**6), x)
```

3.76 $\int \frac{1}{a+b\cos^8(x)}\,dx$

Optimal. Leaf size=245

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] 1/4*arctan(cot(x)*((-a)^(1/4)-b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(
1/4)-b^(1/4))^(1/2)+1/4*arctan(cot(x)*((-a)^(1/4)-I*b^(1/4))^(1/2)/(-a)^(1/
8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)+1/4*arctan(cot(x)*((-a)^(1/4)+I
*b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)+1/4*arc
tan(cot(x)*((-a)^(1/4)+b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+b^
(1/4))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\cot(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^8)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Cot[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt
[(-a)^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Cot[x])/(-
a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[(-a)^(
1/4) + b^(1/4)]*Cot[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + b^(1/4)
]) + ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Cot[x])/(-a)^(5/8)]/(4*(-a)^(3/8)
*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \cot(x)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \cot(x)\right)}{4a} - \text{Subst}\Big(\int$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}} + \frac{\tan^{-1}\Big(}{4(-a)^{3}}$$

Mathematica [C] time = 0.27, size = 172, normalized size = 0.70

$$8\text{RootSum}\left[\#1^8 b+8\#1^7 b+28\#1^6 b+56\#1^5 b+256\#1^4 a+70\#1^4 b+56\#1^3 b+28\#1^2 b+8\#1 b+b\&, \frac{}{\#1^7 b+7}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 + 56*
b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #
1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 +
128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cos(x)^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^8),x, algorithm="giac")
```

```
[Out] integrate(1/(b*cos(x)^8 + a), x)
```

maple [C] time = 0.22, size = 76, normalized size = 0.31

$$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a+b\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^8),x)
```

```
[Out] 1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\cos(x)^8 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^8 + a), x)
```

mupad [B] time = 3.42, size = 216, normalized size = 0.88

$$\sum_{k=1}^{8} \ln\left(\mathrm{root}\left(16777216\,a^7\,b\,d^8 + 16777216\,a^8\,d^8 + 1048576\,a^6\,d^6 + 24576\,a^4\,d^4 + 256\,a^2\,d^2 + 1, d, k\right)^4\,a^5\,b^5\,\left(\mathrm{root}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^8),x)
```

```
[Out] symsum(log(4096*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^
6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^4*a^5*b^5*(64*root(16777216*a^7*
b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 +
1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576
*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(16777
216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^
2*d^2 + 1, d, k), k, 1, 8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cos^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**8),x)
```

```
[Out] Integral(1/(a + b*cos(x)**8), x)
```

3.77 $\int \frac{1}{a-b\cos^5(x)}\,dx$

Optimal. Leaf size=494

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1}$$

```
[Out] 2/5*arctan((a^(1/5)+b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-b^(1/5))^(1/2))/a^(4
/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1
)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2))/a^(4/
5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2)+2/
5*arctan((a^(1/5)+(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(2/5)*
b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(2
/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2)*tan(1/2*x)
/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(
1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(4/5)*b^(1
/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-
(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3213, 2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a^(1/5) + b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - b^(1/5)]])/(5*a
^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt[a
^(1/5) - (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(1/5)*b^(1/5)]])
/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5)*b^
(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1
/5) - (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*S
qrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^(3/5)*b
^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1
/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2*ArcTan[(
Sqrt[a^(1/5) + (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(4/5)*b^(1
/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(4
/5)*b^(1/5)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos\right.}\right.$$
$$=\frac{\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}\cos(x)}}{5a^{4/5}}$$
$$=\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}+\left(\sqrt[5]{a}+\sqrt[5]{b}\right)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}+\left(\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}\right)x^2}\,dx,x\right.}{5a^{4/5}}$$
$$=\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\right.}{5a^{4/5}\sqrt{\sqrt[5]{a}-(}}$$

Mathematica [C] time = 0.18, size = 130, normalized size = 0.26

$$-\frac{8}{5}\mathrm{RootSum}\left[\#1^{10}b+5\#1^8b+10\#1^6b-32\#1^5a+10\#1^4b+5\#1^2b+b\&,\frac{2\#1^3\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\#1^3\log\left(\#1^2-\right.}{\#1^8b+4\#1^6b+6\#1^4b-16\#1^3a+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Cos[x]^5)^(-1),x]
```

```
[Out] (-8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 - 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b
*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 + #
1^2]*#1^3)/(b + 4*b*#1^2 - 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])/5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\cos(x)^5-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*cos(x)^5 - a), x)
```

maple [C] time = 0.10, size = 148, normalized size = 0.30

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{10}+(5a-5b)\_Z^{8}+(10a+10b)\_Z^{6}+(10a-10b)\_Z^{4}+(5a+5b)\_Z^{2}+a-b\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+\_R^{9}b+4\_R^{7}a-4\_R^{7}b+6\_R^{5}a+6\_R^{5}b+4\_R^{3}a-4\_R^{3}b+\_R a+\_R b}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^5),x)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+_R^9*b+4*_R^7*a-4*_R^7*b+6*_R
^5*a+6*_R^5*b+4*_R^3*a-4*_R^3*b+_R*a+_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a+b
)*_Z^10+(5*a-5*b)*_Z^8+(10*a+10*b)*_Z^6+(10*a-10*b)*_Z^4+(5*a+5*b)*_Z^2+a-b
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\cos(x)^5 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^5),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^5 - a), x)
```

mupad [B] time = 7.83, size = 1518, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^5),x)
```

```
[Out] symsum(log(-(10995116277760*b^7*(a + b)*(56*root(9765625*a^8*b^2*d^10 - 976
5625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)*a - 7*cot(x/2) + root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^
10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d,
k)*b + 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 + 225000*ro
ot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*
d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 3875000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 25000000*root(9765625*a^8*b^2*d^10 -
 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*
a^2*d^2 - 1, d, k)^9*a^9 - 735*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1
0 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k
)^2*a^2*cot(x/2) - 28875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 19
53125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^
4*cot(x/2) - 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125
*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^6*cot
(x/2) - 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^8*cot(x/2
) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b + 100000*root
(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^
6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b + 4000000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b + 50000000*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^9*a^8*b + 125000*root(9765625*a^8*b^2*d^10 - 9765625*a
```

```
^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 -
1, d, k)^7*a^5*b^2 + 25000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^9*a^7*b^2 - 35*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a*b*cot(x/2
) - 7000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^3*b*cot(x/2) - 3
50000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156
250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b*cot(x/2) - 5000
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 15625
0*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b*cot(x/2) - 3125*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*cot(x/2) - 1718750*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*cot(x/2)))/cot(x/2))
*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\cos^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**5),x)
```

```
[Out] Integral(1/(a - b*cos(x)**5), x)
```

3.78 $\int \frac{1}{a-b\cos^6(x)}\,dx$

Optimal. Leaf size=175

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] -1/3*arctan(cot(x)*(a^(1/3)-b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/3
))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)/a^(1/6))/a^(5
/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)-(-1)^(2/3
)*b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3211, 3181, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^6)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
 b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3
*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) - (-1)^
(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3
)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \cot(x)\right)}{3a} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \cot(x)\right)}{3a} - \text{Subst}\left(\int \right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.18, size = 146, normalized size = 0.83

$$-\frac{8}{3}\text{RootSum}\left[\#1^6b+6\#1^5b+15\#1^4b-64\#1^3a+20\#1^3b+15\#1^2b+6\#1b+b\&, \frac{2\#1^2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^2 l}{\#1^5b+5\#1^4b+10\#1^3b-32\#}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cos[x]^6)^(-1),x]
```

```
[Out] (-8*RootSum[b + 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*
Cos[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 - 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 +
 5*b*#1^4 + b*#1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^6),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.43, size = 62, normalized size = 0.35

$$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+3a\_Z^2+a-b\right)} \frac{\left(\_R^4+2\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^5+2\_R^3+\_R}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^6),x)
```

```
[Out] 1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+3
*_Z^4*a+3*_Z^2*a+a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\cos(x)^6 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^6),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^6 - a), x)
```

mupad [B] time = 3.12, size = 184, normalized size = 1.05

$$\sum_{k=1}^{6} \ln\left(-\mathrm{root}\left(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k\right)^2\,a^3\,b^3\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6 - 4665\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^6),x)
```

```
[Out] symsum(log(-36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^
2*d^2 - 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*
a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 - 46656
*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root(46656*
a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\cos^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**6),x)
```

```
[Out] Integral(1/(a - b*cos(x)**6), x)
```

3.79 $\int \frac{1}{a-b\cos^8(x)}\,dx$

Optimal. Leaf size=213

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

```
[Out] -1/4*arctan(cot(x)*(a^(1/4)-b^(1/4))^(1/2)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/4
))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)-I*b^(1/4))^(1/2)/a^(1/8))/a^(7/8)/(a^(1
/4)-I*b^(1/4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)+I*b^(1/4))^(1/2)/a^(1/8))/a
^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)+b^(1/4))^(1/2)/
a^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3211, 3181, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
 b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7/8)*
Sqrt[a^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Cot[x])/a^(1
/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + b^(1/4)
]*Cot[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}dx, x, \cot(x)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}dx, x, \cot(x)\right)}{4a} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\,\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\,\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\,\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\sqrt{\right.}{4a^{7/8}\sqrt{}}$$

Mathematica [C] time = 0.22, size = 172, normalized size = 0.81

$$-8\text{RootSum}\left[\#1^8 b + 8\#1^7 b + 28\#1^6 b + 56\#1^5 b - 256\#1^4 a + 70\#1^4 b + 56\#1^3 b + 28\#1^2 b + 8\#1 b + b\&, \frac{}{\#1^7 b +}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Cos[x]^8)^(-1),x]
```

```
[Out] -8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 + 56
*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
#1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 -
 128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\cos(x)^8 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^8),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*cos(x)^8 - a), x)
```

maple [C] time = 0.22, size = 78, normalized size = 0.37

$$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a-b\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^8),x)
```

```
[Out] 1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\cos(x)^8 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^8),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^8 - a), x)
```

mupad [B] time = 3.42, size = 216, normalized size = 1.01

$$\sum_{k=1}^{8} \ln\left(-\mathrm{root}\left(16777216\, a^7\, b\, d^8 - 16777216\, a^8\, d^8 - 1048576\, a^6\, d^6 - 24576\, a^4\, d^4 - 256\, a^2\, d^2 - 1, d, k\right)^4 a^5\, b^5\, \left(\mathrm{root}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^8),x)
```

```
[Out] symsum(log(-4096*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d
^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^4*a^5*b^5*(64*root(16777216*a^7
*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 -
 1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 104857
6*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root(1677
7216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a
^2*d^2 - 1, d, k), k, 1, 8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\cos^8(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**8),x)
```

```
[Out] Integral(1/(a - b*cos(x)**8), x)
```

3.80 $\int \frac{1}{1+\cos^5(x)}\,dx$

Optimal. Leaf size=223

$$\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{\sin(x)}{5(\cos(x)+1)}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)}{1-(-1)}}}{}$$

```
[Out] 1/5*sin(x)/(1+cos(x))-2/5*arctanh(tan(1/2*x)/((-1+(-1)^(1/5))/(1+(-1)^(1/5)
))^(1/2))/(-1+(-1)^(2/5))^(1/2)+2/5*arctan(((1-(-1)^(4/5))/(1+(-1)^(4/5)))^
(1/2)*tan(1/2*x))/(1+(-1)^(3/5))^(1/2)-2/5*arctanh(((-1-(-1)^(3/5))/(1-(-1)
^(3/5)))^(1/2)*tan(1/2*x))*((-1-(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)/(1+(-1)^(
3/5))+2/5*arctan(((1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2)*tan(1/2*x))/(1-(-1)^
(4/5))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3213, 2648, 2659, 208, 205}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{\sin(x)}{5(\cos(x)+1)}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)}{1-(-1)}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[Sqrt[(1 - (-1)^(2/5))/(1 + (-1)^(2/5))]*Tan[x/2]])/(5*Sqrt[1 - (-
1)^(4/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(4/5))/(1 + (-1)^(4/5))]*Tan[x/2]])/(
5*Sqrt[1 + (-1)^(3/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(1/5))/(1 +
(-1)^(1/5)))]])/(5*Sqrt[-1 + (-1)^(2/5)]) - (2*Sqrt[-((1 + (-1)^(3/5))/(1 -
 (-1)^(3/5)))]*ArcTanh[Sqrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*Tan[x/2]]
)/(5*(1 + (-1)^(3/5))) + Sin[x]/(5*(1 + Cos[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1+\cos^5(x)}\,dx = \int \left(-\frac{1}{5(-1-\cos(x))} - \frac{1}{5\left(-1+\sqrt[5]{-1}\cos(x)\right)} - \frac{1}{5\left(-1-(-1)^{2/5}\cos(x)\right)} - \frac{1}{5\left(-1+(-1)^{3/5}\cos\right.} \cdots$$

$$= -\left(\frac{1}{5}\int \frac{1}{-1-\cos(x)}\,dx\right) - \frac{1}{5}\int \frac{1}{-1+\sqrt[5]{-1}\cos(x)}\,dx - \frac{1}{5}\int \frac{1}{-1-(-1)^{2/5}\cos(x)}\,dx - \frac{1}{5}\int \cdots$$

$$= \frac{\sin(x)}{5(1+\cos(x))} - \frac{2}{5}\mathrm{Subst}\left(\int \frac{1}{-1+\sqrt[5]{-1}+\left(-1-\sqrt[5]{-1}\right)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{2}{5}\mathrm{Subst}\left(\int \cdots\right.$$

$$= \frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}} - \frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}} - \frac{2\sqrt{-\cdots}}{\cdots}$$

Mathematica [C] time = 0.13, size = 378, normalized size = 1.70

$$\frac{1}{5}\tan\left(\frac{x}{2}\right) - \frac{1}{10}\mathrm{RootSum}\left[\#1^8 - 2\#1^7 + 8\#1^6 - 14\#1^5 + 30\#1^4 - 14\#1^3 + 8\#1^2 - 2\#1 + 1\&, \frac{2\#1^6\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^5)^(-1),x]
```

```
[Out] -1/10*RootSum[1 - 2*#1 + 8*#1^2 - 14*#1^3 + 30*#1^4 - 14*#1^5 + 8*#1^6 - 2*
#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos[x]*#1 + #
1^2] - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + (4*I)*Log[1 - 2*Cos[x]*#1 + #1^2
]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 - 2*Cos[x]*#1 +
#1^2]*#1^2 - 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 + (40*I)*Log[1 - 2*Cos[x]
*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (15*I)*Log[1 - 2*
Cos[x]*#1 + #1^2]*#1^4 - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^5 + (4*I)*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 - I*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^6)/(-1 + 8*#1 - 21*#1^2 + 60*#1^3 - 35*#1^4 + 24*#
1^5 - 7*#1^6 + 4*#1^7) & ] + Tan[x/2]/5
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.07, size = 62, normalized size = 0.28

$$\frac{\tan\left(\frac{x}{2}\right)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^{8}+10\_Z^{4}+1\right)}\frac{\left(5\_R^{6}+5\_R^{4}+5\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}+\_R^{3}}\right)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^5),x)
```

```
[Out] 1/5*tan(1/2*x)+1/50*sum((5*_R^6+5*_R^4+5*_R^2+1)/(_R^7+_R^3)*ln(tan(1/2*x)-
_R),_R=RootOf(5*_Z^8+10*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^5),x, algorithm="maxima")
```

```
[Out] -1/5*(5*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(-2/5*((cos(7*x) - 4*
cos(6*x) + 15*cos(5*x) - 40*cos(4*x) + 15*cos(3*x) - 4*cos(2*x) + cos(x))*c
os(8*x) + (16*cos(6*x) - 44*cos(5*x) + 110*cos(4*x) - 44*cos(3*x) + 16*cos(
2*x) - 4*cos(x) + 1)*cos(7*x) - 2*cos(7*x)^2 + 4*(44*cos(5*x) - 110*cos(4*x
) + 44*cos(3*x) - 16*cos(2*x) + 4*cos(x) - 1)*cos(6*x) - 32*cos(6*x)^2 + (1
010*cos(4*x) - 420*cos(3*x) + 176*cos(2*x) - 44*cos(x) + 15)*cos(5*x) - 210
*cos(5*x)^2 + 10*(101*cos(3*x) - 44*cos(2*x) + 11*cos(x) - 4)*cos(4*x) - 12
00*cos(4*x)^2 + (176*cos(2*x) - 44*cos(x) + 15)*cos(3*x) - 210*cos(3*x)^2 +
 4*(4*cos(x) - 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (sin(7*x) - 4*sin
(6*x) + 15*sin(5*x) - 40*sin(4*x) + 15*sin(3*x) - 4*sin(2*x) + sin(x))*sin(
8*x) + 2*(8*sin(6*x) - 22*sin(5*x) + 55*sin(4*x) - 22*sin(3*x) + 8*sin(2*x)
 - 2*sin(x))*sin(7*x) - 2*sin(7*x)^2 + 8*(22*sin(5*x) - 55*sin(4*x) + 22*si
n(3*x) - 8*sin(2*x) + 2*sin(x))*sin(6*x) - 32*sin(6*x)^2 + 2*(505*sin(4*x)
- 210*sin(3*x) + 88*sin(2*x) - 22*sin(x))*sin(5*x) - 210*sin(5*x)^2 + 10*(1
01*sin(3*x) - 44*sin(2*x) + 11*sin(x))*sin(4*x) - 1200*sin(4*x)^2 + 44*(4*s
in(2*x) - sin(x))*sin(3*x) - 210*sin(3*x)^2 - 32*sin(2*x)^2 + 16*sin(2*x)*s
in(x) - 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) - 8*cos(6*x) + 14*cos(5*x) - 30
*cos(4*x) + 14*cos(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(8*x) - cos(8*x)^2
+ 4*(8*cos(6*x) - 14*cos(5*x) + 30*cos(4*x) - 14*cos(3*x) + 8*cos(2*x) - 2*
cos(x) + 1)*cos(7*x) - 4*cos(7*x)^2 + 16*(14*cos(5*x) - 30*cos(4*x) + 14*co
s(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(6*x) - 64*cos(6*x)^2 + 28*(30*cos(4
*x) - 14*cos(3*x) + 8*cos(2*x) - 2*cos(x) + 1)*cos(5*x) - 196*cos(5*x)^2 +
60*(14*cos(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 28
*(8*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - 196*cos(3*x)^2 + 16*(2*cos(x) - 1)*
cos(2*x) - 64*cos(2*x)^2 - 4*cos(x)^2 + 4*(sin(7*x) - 4*sin(6*x) + 7*sin(5*
x) - 15*sin(4*x) + 7*sin(3*x) - 4*sin(2*x) + sin(x))*sin(8*x) - sin(8*x)^2
+ 8*(4*sin(6*x) - 7*sin(5*x) + 15*sin(4*x) - 7*sin(3*x) + 4*sin(2*x) - sin(
x))*sin(7*x) - 4*sin(7*x)^2 + 32*(7*sin(5*x) - 15*sin(4*x) + 7*sin(3*x) - 4
*sin(2*x) + sin(x))*sin(6*x) - 64*sin(6*x)^2 + 56*(15*sin(4*x) - 7*sin(3*x)
 + 4*sin(2*x) - sin(x))*sin(5*x) - 196*sin(5*x)^2 + 120*(7*sin(3*x) - 4*sin
(2*x) + sin(x))*sin(4*x) - 900*sin(4*x)^2 + 56*(4*sin(2*x) - sin(x))*sin(3*
x) - 196*sin(3*x)^2 - 64*sin(2*x)^2 + 32*sin(2*x)*sin(x) - 4*sin(x)^2 + 4*c
os(x) - 1), x) - 2*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

mupad [B] time = 2.78, size = 535, normalized size = 2.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5 + 1),x)
```

```
[Out] tan(x/2)/5 + 2*atanh((603979776*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1
220703125 - (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5 - 1
)^(1/2))/1220703125 - 301989888/1220703125)) + (268435456*5^(1/2)*tan(x/2)*
(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1
/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/122070312
5 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 301989888/122070312
5)))*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((603979776*t
an(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((3355443
2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5^(1/2))/122
0703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + 301989888/122
0703125)) + (268435456*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1
/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220
703125 + (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(
1/2))/1220703125 + 301989888/1220703125)))*((- (2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2) - 2*atanh((603979776*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/12
20703125 - (134217728*5^(1/2))/1220703125 - (67108864*((2*5^(1/2))/5 - 1)^(
1/2))/1220703125 + 301989888/1220703125)) - (268435456*5^(1/2)*tan(x/2)*(-
((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(
(2*5^(1/2))/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/1220703125 - (67
108864*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 + 301989888/1220703125)))*(- (
(2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) + 2*atanh((603979776*tan(x/2)*(((
2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*((2
*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5^(1/2))/1220703125 - (6710
8864*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 301989888/1220703125)) - (2684
35456*5^(1/2)*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140
625*((33554432*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5
^(1/2))/1220703125 - (67108864*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 3019
89888/1220703125)))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**5),x)
```

```
[Out] Timed out
```

3.81 $\int \frac{1}{1+\cos^6(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] 1/6*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)+1/3*arctan(tan(x)/(1-(-1)^(1/3))^(1/
2))/(1-(-1)^(1/3))^(1/2)+1/3*arctan(tan(x)/(1+(-1)^(2/3))^(1/2))/(1+(-1)^(2
/3))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 203}

$$\frac{x}{3\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1+(-1)^{2/3}}}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^6)^(-1),x]
```

```
[Out] x/(3*Sqrt[2]) - ArcTan[Sqrt[1 - (-1)^(1/3)]*Cot[x]]/(3*Sqrt[1 - (-1)^(1/3)]
) - ArcTan[Sqrt[1 + (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 + (-1)^(2/3)]) - ArcTan[(
Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\cos^6(x)}\,dx=\frac{1}{3}\int \frac{1}{1+\cos^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\,\cos^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\cos^2(x)}\,dx$$

$$=-\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[3]{-1}\right)x^2}\,dx,x,\cot(x)\right)-\frac{1}{3}$$

$$=\frac{x}{3\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1+(-1)^{2/3}}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.17, size = 79, normalized size = 0.95

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)+\log(2-\sin(2x))-\log(\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2
]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + Log[2 - Sin[2*x]] - Log[2 +
 Sin[2*x]])/12
```

fricas [B] time = 3.09, size = 138, normalized size = 1.66

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(}{4\cos(x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^6),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1
)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^
2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*si
n(x))) - 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/24*log(-c
os(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

giac [B] time = 0.38, size = 185, normalized size = 2.23

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)+\cos(2x)-2\sin(2x)+1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)-\sin(2x)+2}\right)\right)+\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\mathrm{co}}{\sqrt{3}\cos(2x)+\sqrt{3}-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(sq
rt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x +
arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x) +
sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt(2)*
sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) - 1/12*l
og(tan(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

maple [A] time = 0.07, size = 73, normalized size = 0.88

$$-\frac{\ln\left(\tan^2(x)+\tan(x)+1\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{6}+\frac{\ln\left(\tan^2(x)-\tan(x)+1\right)}{12}+\frac{\sqrt{3}\ a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^6),x)
```

```
[Out] -1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1)*3^(1/2))+1/
6*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)+1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)
*arctan(1/3*(2*tan(x)-1)*3^(1/2))
```

maxima [A] time = 0.78, size = 72, normalized size = 0.87

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right)+\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-\frac{1}{12}\log\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/12*log(ta
n(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

mupad [B] time = 2.39, size = 99, normalized size = 1.19

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{6}+\mathrm{atan}\left(\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)-\mathrm{atan}\left(-\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6 + 1),x)
```

```
[Out] atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) - atan((tan(x)*
1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) + (2^(1/2)*atan((2^(1/2)*tan
(x))/2))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6) +
pi*(3^(1/2)/6 + 1i/6)))/pi
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**6),x)
```

```
[Out] Timed out
```

3.82 $\int \frac{1}{1+\cos^8(x)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\frac{\tan^{-1}\left(\sqrt{1+(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

```
[Out] -1/4*arctan(cot(x)*(1-(-1)^(1/4))^(1/2))/(1-(-1)^(1/4))^(1/2)-1/4*arctan(co
t(x)*(1+(-1)^(1/4))^(1/2))/(1+(-1)^(1/4))^(1/2)-1/4*arctan(cot(x)*(1-(-1)^(
3/4))^(1/2))/(1-(-1)^(3/4))^(1/2)-1/4*arctan(cot(x)*(1+(-1)^(3/4))^(1/2))/(
1+(-1)^(3/4))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 203}

$$-\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\frac{\tan^{-1}\left(\sqrt{1+(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^8)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[1 - (-1)^(1/4)]*Cot[x]]/(4*Sqrt[1 - (-1)^(1/4)]) - ArcTan[Sqrt
[1 + (-1)^(1/4)]*Cot[x]]/(4*Sqrt[1 + (-1)^(1/4)]) - ArcTan[Sqrt[1 - (-1)^(3
/4)]*Cot[x]]/(4*Sqrt[1 - (-1)^(3/4)]) - ArcTan[Sqrt[1 + (-1)^(3/4)]*Cot[x]]
/(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\cos^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\cos^2(x)}\,dx + \frac{1}{4}\int$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \cot(x)\right)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x, \text{cc}\right.$$

$$= -\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}$$

Mathematica [C] time = 0.15, size = 141, normalized size = 1.09

$$8\text{RootSum}\left[\#1^8+8\#1^7+28\#1^6+56\#1^5+326\#1^4+56\#1^3+28\#1^2+8\#1+1\&, \frac{2\#1^3\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#}{\#1^7+7\#1^6+21\#1^5+35\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[1 + 8*#1 + 28*#1^2 + 56*#1^3 + 326*#1^4 + 56*#1^5 + 28*#1^6 + 8*#
1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos[2
*x]*#1 + #1^2]*#1^3)/(1 + 7*#1 + 21*#1^2 + 163*#1^3 + 35*#1^4 + 21*#1^5 + 7
*#1^6 + #1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^8),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.07, size = 67, normalized size = 0.52

$$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+2\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^8),x)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=Roo
tOf(_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(x)^8 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(cos(x)^8 + 1), x)
```

mupad [B] time = 3.11, size = 1025, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^8 + 1),x)
```

```
[Out] atan((tan(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(
1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (2^(1/2)*tan(
x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^
(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (tan(x)*(2*2^(1/2) - 3)
^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) -
 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) + (2^(1/2)*tan(x)*(2*
2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*
(2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1))*((2*2^(1
/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(x)*(- (2*2^(1/2) - 3)^(1/2
)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 - (
2*2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(- (2*2^(1/2) - 3)^(1/2)/128 -
1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 - (2*2^(1/2
) - 3)^(1/2) - 1) + (tan(x)*(2*2^(1/2) - 3)^(1/2)*(- (2*2^(1/2) - 3)^(1/2)/
128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 - (2*
2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(2*2^(1/2) - 3)^(1/2)*(- (2*2^(1/
2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2
^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1))*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128
)^(1/2)*2i + atan((tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)
/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2)
 + 1) + (2^(1/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/(
(2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2) +
 1) + (tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/12
8)^(1/2)*8i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2*2^(1/2
) - 3)^(1/2) + 1) + (2^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2)
 - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2
^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2) + 1))*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1
/128)^(1/2)*2i - atan((tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8
i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/
2) - 1) + (2^(1/2)*tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/(
(2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2) -
 1) - (tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)
^(1/2)*8i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 + (- 2*2^(1/2)
- 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3
)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/
2)/2 + (- 2*2^(1/2) - 3)^(1/2) - 1))*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^
(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**8),x)

[Out] Timed out
```

3.83 $\int \frac{1}{1-\cos^5(x)}\,dx$

Optimal. Leaf size=205

$$\frac{2\tan^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{\sin(x)}{5(1-\cos(x))}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\tanh^{-1}\left(\sqrt{-\right.}{5\sqrt{-1}}$$

```
[Out] -1/5*sin(x)/(1-cos(x))+2/5*arctan(((1-(-1)^(3/5))/(1+(-1)^(3/5)))^(1/2)*tan
(1/2*x))/(1+(-1)^(1/5))^(1/2)+2/5*arctan(((1-(-1)^(1/5))/(1+(-1)^(1/5)))^(1
/2)*tan(1/2*x))/(1-(-1)^(2/5))^(1/2)+2/5*arctanh(((-1-(-1)^(4/5))/(1-(-1)^(
4/5)))^(1/2)*tan(1/2*x))/(-1-(-1)^(3/5))^(1/2)-2/5*arctanh(tan(1/2*x)/((-1+
(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2))/(-1+(-1)^(4/5))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3213, 2648, 2659, 205, 208}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{\sin(x)}{5(1-\cos(x))}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\tanh^{-1}\left(\sqrt{-\right.}{5\sqrt{-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[Sqrt[(1 - (-1)^(1/5))/(1 + (-1)^(1/5))]*Tan[x/2]])/(5*Sqrt[1 - (-
1)^(2/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(3/5))/(1 + (-1)^(3/5))]*Tan[x/2]])/(
5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(2/5))/(1 +
(-1)^(2/5)))]])/(5*Sqrt[-1 + (-1)^(4/5)]) + (2*ArcTanh[Sqrt[-((1 + (-1)^(4/
5))/(1 - (-1)^(4/5)))]*Tan[x/2]])/(5*Sqrt[-1 - (-1)^(3/5)]) - Sin[x]/(5*(1
- Cos[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1-\cos^5(x)}\,dx = \int \left(\frac{1}{5(1-\cos(x))}+\frac{1}{5\left(1+\sqrt[5]{-1}\cos(x)\right)}+\frac{1}{5\left(1-(-1)^{2/5}\cos(x)\right)}+\frac{1}{5\left(1+(-1)^{3/5}\cos(x)\right)}\right.$$

$$=\frac{1}{5}\int \frac{1}{1-\cos(x)}\,dx+\frac{1}{5}\int \frac{1}{1+\sqrt[5]{-1}\cos(x)}\,dx+\frac{1}{5}\int \frac{1}{1-(-1)^{2/5}\cos(x)}\,dx+\frac{1}{5}\int \frac{}{1+(-1}$$

$$=-\frac{\sin(x)}{5(1-\cos(x))}+\frac{2}{5}\mathrm{Subst}\left(\int \frac{1}{1+\sqrt[5]{-1}+\left(1-\sqrt[5]{-1}\right)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)+\frac{2}{5}\mathrm{Subst}\left(\int \frac{}{1-}\right.$$

$$=\frac{2\tan^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{-1+(-1)^{4/5}}}+\frac{2}{}$$

Mathematica [C] time = 0.12, size = 378, normalized size = 1.84

$$-\frac{1}{5}\cot\left(\frac{x}{2}\right)+\frac{1}{10}\mathrm{RootSum}\left[\#1^8+2\#1^7+8\#1^6+14\#1^5+30\#1^4+14\#1^3+8\#1^2+2\#1+1\&,\frac{2\#1^6\tan^{-1}\left(\frac{\sin(}{\cos(x}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^5)^(-1),x]
```

```
[Out] -1/5*Cot[x/2] + RootSum[1 + 2*#1 + 8*#1^2 + 14*#1^3 + 30*#1^4 + 14*#1^5 + 8
*#1^6 + 2*#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos
[x]*#1 + #1^2] + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - (4*I)*Log[1 - 2*Cos[x]
*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 - 2*Co
s[x]*#1 + #1^2]*#1^2 + 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - (40*I)*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (15*I)*
Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^5 - (4
*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6
- I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(1 + 8*#1 + 21*#1^2 + 60*#1^3 + 35*#1
^4 + 24*#1^5 + 7*#1^6 + 4*#1^7) & ]/10
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.08, size = 62, normalized size = 0.30

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+10\_Z^4+5\right)}\frac{\left(\_R^6+5\_R^4+5\_R^2+5\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^7+5\_R^3}\right)}{10}-\frac{1}{5\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^5),x)
```

```
[Out] 1/10*sum((_R^6+5*_R^4+5*_R^2+5)/(_R^7+5*_R^3)*ln(tan(1/2*x)-_R),_R=RootOf(_
Z^8+10*_Z^4+5))-1/5/tan(1/2*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^5),x, algorithm="maxima")
```

```
[Out] 1/5*(5*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(2/5*((cos(7*x) + 4*co
s(6*x) + 15*cos(5*x) + 40*cos(4*x) + 15*cos(3*x) + 4*cos(2*x) + cos(x))*cos
(8*x) + (16*cos(6*x) + 44*cos(5*x) + 110*cos(4*x) + 44*cos(3*x) + 16*cos(2*
x) + 4*cos(x) + 1)*cos(7*x) + 2*cos(7*x)^2 + 4*(44*cos(5*x) + 110*cos(4*x)
+ 44*cos(3*x) + 16*cos(2*x) + 4*cos(x) + 1)*cos(6*x) + 32*cos(6*x)^2 + (101
0*cos(4*x) + 420*cos(3*x) + 176*cos(2*x) + 44*cos(x) + 15)*cos(5*x) + 210*c
os(5*x)^2 + 10*(101*cos(3*x) + 44*cos(2*x) + 11*cos(x) + 4)*cos(4*x) + 1200
*cos(4*x)^2 + (176*cos(2*x) + 44*cos(x) + 15)*cos(3*x) + 210*cos(3*x)^2 + 4
*(4*cos(x) + 1)*cos(2*x) + 32*cos(2*x)^2 + 2*cos(x)^2 + (sin(7*x) + 4*sin(6
*x) + 15*sin(5*x) + 40*sin(4*x) + 15*sin(3*x) + 4*sin(2*x) + sin(x))*sin(8*
x) + 2*(8*sin(6*x) + 22*sin(5*x) + 55*sin(4*x) + 22*sin(3*x) + 8*sin(2*x) +
 2*sin(x))*sin(7*x) + 2*sin(7*x)^2 + 8*(22*sin(5*x) + 55*sin(4*x) + 22*sin(
3*x) + 8*sin(2*x) + 2*sin(x))*sin(6*x) + 32*sin(6*x)^2 + 2*(505*sin(4*x) +
210*sin(3*x) + 88*sin(2*x) + 22*sin(x))*sin(5*x) + 210*sin(5*x)^2 + 10*(101
*sin(3*x) + 44*sin(2*x) + 11*sin(x))*sin(4*x) + 1200*sin(4*x)^2 + 44*(4*sin
(2*x) + sin(x))*sin(3*x) + 210*sin(3*x)^2 + 32*sin(2*x)^2 + 16*sin(2*x)*sin
(x) + 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) + 8*cos(6*x) + 14*cos(5*x) + 30*c
os(4*x) + 14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(8*x) + cos(8*x)^2 +
4*(8*cos(6*x) + 14*cos(5*x) + 30*cos(4*x) + 14*cos(3*x) + 8*cos(2*x) + 2*co
s(x) + 1)*cos(7*x) + 4*cos(7*x)^2 + 16*(14*cos(5*x) + 30*cos(4*x) + 14*cos(
3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(6*x) + 64*cos(6*x)^2 + 28*(30*cos(4*x
) + 14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(5*x) + 196*cos(5*x)^2 + 60
*(14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(4*x) + 900*cos(4*x)^2 + 28*(
8*cos(2*x) + 2*cos(x) + 1)*cos(3*x) + 196*cos(3*x)^2 + 16*(2*cos(x) + 1)*co
s(2*x) + 64*cos(2*x)^2 + 4*cos(x)^2 + 4*(sin(7*x) + 4*sin(6*x) + 7*sin(5*x)
 + 15*sin(4*x) + 7*sin(3*x) + 4*sin(2*x) + sin(x))*sin(8*x) + sin(8*x)^2 +
8*(4*sin(6*x) + 7*sin(5*x) + 15*sin(4*x) + 7*sin(3*x) + 4*sin(2*x) + sin(x)
)*sin(7*x) + 4*sin(7*x)^2 + 32*(7*sin(5*x) + 15*sin(4*x) + 7*sin(3*x) + 4*s
in(2*x) + sin(x))*sin(6*x) + 64*sin(6*x)^2 + 56*(15*sin(4*x) + 7*sin(3*x) +
 4*sin(2*x) + sin(x))*sin(5*x) + 196*sin(5*x)^2 + 120*(7*sin(3*x) + 4*sin(2
*x) + sin(x))*sin(4*x) + 900*sin(4*x)^2 + 56*(4*sin(2*x) + sin(x))*sin(3*x)
 + 196*sin(3*x)^2 + 64*sin(2*x)^2 + 32*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos
(x) + 1), x) - 2*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.45, size = 403, normalized size = 1.97

$$2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\sqrt{5}\,\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{-\frac{2\sqrt{5}}{5}-1}+2\sqrt{5}-10\sqrt{-\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-2\,\mathrm{atanh}\left(\frac{50}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^5 - 1),x)
```

```
[Out] 2*atanh((50*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^(
1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(-
(2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) - 5))
*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((50*tan(x/2)*(- (-
 (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^(1/2)*tan(x/2)*(- (- (2*5
^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/
2) - 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) + 5))*(- (- (2*5^(1/2))/5 -
 1)^(1/2)/50 - 1/50)^(1/2) - cot(x/2)/5 + 2*atanh((50*tan(x/2)*(- ((2*5^(1/
2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) + 20*5^(1/2)*tan(x/2)*(- ((2*5^(1/2))/5 -
 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2) - 2*5^(1/2
) + 10*((2*5^(1/2))/5 - 1)^(1/2) - 5))*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/
50)^(1/2) - 2*atanh((50*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
) + 20*5^(1/2)*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(
1/2)*((2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) + 10*((2*5^(1/2))/5 - 1)^(1/2) +
 5))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**5),x)
```

```
[Out] Timed out
```

3.84 $\int \frac{1}{1-\cos^6(x)}\,dx$

Optimal. Leaf size=71

$$-\frac{\cot(x)}{3}-\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

```
[Out] -1/3*cot(x)-1/3*arctan(cot(x)*(1+(-1)^(1/3))^(1/2))/(1+(-1)^(1/3))^(1/2)-1/
3*arctan(cot(x)*(1-(-1)^(2/3))^(1/2))/(1-(-1)^(2/3))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 203, 3175, 3767, 8}

$$-\frac{\cot(x)}{3}-\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^6)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[1 + (-1)^(1/3)]*Cot[x]]/(3*Sqrt[1 + (-1)^(1/3)]) - ArcTan[Sqrt
[1 - (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 - (-1)^(2/3)]) - Cot[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cos^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1-\cos^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\,\cos^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cos^2(x)}\,dx\\ &= \frac{1}{3}\int \csc^2(x)\,dx-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[3]{-1}\right)x^2}\,dx,x,\cot(x)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1-(-1\right.}\right.\\ &= -\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{1}{3}\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{\cot(x)}{3}\end{aligned}$$

Mathematica [C] time = 0.29, size = 117, normalized size = 1.65

$$\frac{\sin(x)(8\cos(2x)+\cos(4x)+15)\left(6\cos(x)+i\sqrt[4]{-3}\left(\sqrt{3}+3i\right)\sin(x)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(\sqrt{3}-i\right)\tan(x)\right)+\sqrt[4]{-3}\,(\cdots\right.}{144\left(\cos^6(x)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^6)^(-1),x]
```

```
[Out] ((15 + 8*Cos[2*x] + Cos[4*x])*Sin[x]*(6*Cos[x] + I*(-3)^(1/4)*(3*I + Sqrt[3
])*ArcTan[((-1/3)^(1/4)*(-I + Sqrt[3])*Tan[x])/2]*Sin[x] + (-3)^(1/4)*(-3*I
 + Sqrt[3])*ArcTan[((-1)^(3/4)*(I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Sin[x]))/
(144*(-1 + Cos[x]^6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.54, size = 199, normalized size = 2.80

$$\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-\arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan(x)\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}\right)\right)\sqrt{6\sqrt{3}+9}+\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}\left(\cdots\right.\right.}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^6),x, algorithm="giac")
```

```
[Out] 1/18*(pi*floor(x/pi + 1/2) - arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(2
)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi*floor(x
/pi + 1/2) + arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(2)) - 4*tan(x))/(
sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) - 1/36*sqrt(6*sqrt(3) - 9)*log(1/2
```

```
*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x) + tan(x)^2 + sqrt(3)) + 1/36*sq
rt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x) + tan
(x)^2 + sqrt(3)) - 1/3/tan(x)
```

maple [B] time = 0.24, size = 233, normalized size = 3.28

$$-\frac{\sqrt{3}\,\sqrt{2\sqrt{3}-3}\,\ln\left(\tan^{2}(x)+\tan(x)\sqrt{2\sqrt{3}-3}+\sqrt{3}\right)}{36}+\frac{\arctan\left(\frac{2\tan(x)+\sqrt{2\sqrt{3}-3}}{\sqrt{2\sqrt{3}+3}}\right)}{3\sqrt{2\sqrt{3}+3}}+\frac{\arctan\left(\frac{2\tan(x)+\sqrt{2\sqrt{3}-3}}{\sqrt{2\sqrt{3}+3}}\right)\sqrt{}}{6\sqrt{2\sqrt{3}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^6),x)
```

```
[Out] -1/36*3^(1/2)*(2*3^(1/2)-3)^(1/2)*ln(tan(x)^2+tan(x)*(2*3^(1/2)-3)^(1/2)+3^
(1/2))+1/3/(2*3^(1/2)+3)^(1/2)*arctan((2*tan(x)+(2*3^(1/2)-3)^(1/2))/(2*3^(
1/2)+3)^(1/2))+1/6/(2*3^(1/2)+3)^(1/2)*arctan((2*tan(x)+(2*3^(1/2)-3)^(1/2)
)/(2*3^(1/2)+3)^(1/2))*3^(1/2)+1/36*3^(1/2)*(2*3^(1/2)-3)^(1/2)*ln(tan(x)^2
-tan(x)*(2*3^(1/2)-3)^(1/2)+3^(1/2))+1/3/(2*3^(1/2)+3)^(1/2)*arctan((2*tan(
x)-(2*3^(1/2)-3)^(1/2))/(2*3^(1/2)+3)^(1/2))+1/6/(2*3^(1/2)+3)^(1/2)*arctan
((2*tan(x)-(2*3^(1/2)-3)^(1/2))/(2*3^(1/2)+3)^(1/2))*3^(1/2)-1/3/tan(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^6),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(1/3*((cos(3*x)
+ 4*cos(2*x) + cos(x))*cos(4*x) + (14*cos(2*x) + 4*cos(x) + 1)*cos(3*x) + 2
*cos(3*x)^2 + 2*(7*cos(x) + 2)*cos(2*x) + 24*cos(2*x)^2 + 2*cos(x)^2 + (sin
(3*x) + 4*sin(2*x) + sin(x))*sin(4*x) + 2*(7*sin(2*x) + 2*sin(x))*sin(3*x)
+ 2*sin(3*x)^2 + 24*sin(2*x)^2 + 14*sin(2*x)*sin(x) + 2*sin(x)^2 + cos(x))/
(2*(2*cos(3*x) + 6*cos(2*x) + 2*cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 4*(6*co
s(2*x) + 2*cos(x) + 1)*cos(3*x) + 4*cos(3*x)^2 + 12*(2*cos(x) + 1)*cos(2*x)
 + 36*cos(2*x)^2 + 4*cos(x)^2 + 4*(sin(3*x) + 3*sin(2*x) + sin(x))*sin(4*x)
 + sin(4*x)^2 + 8*(3*sin(2*x) + sin(x))*sin(3*x) + 4*sin(3*x)^2 + 36*sin(2*
x)^2 + 24*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1), x) - 3*(cos(2*x)^2
+ sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(-1/3*((cos(3*x) - 4*cos(2*x) + cos
(x))*cos(4*x) + (14*cos(2*x) - 4*cos(x) + 1)*cos(3*x) - 2*cos(3*x)^2 + 2*(7
*cos(x) - 2)*cos(2*x) - 24*cos(2*x)^2 - 2*cos(x)^2 + (sin(3*x) - 4*sin(2*x)
 + sin(x))*sin(4*x) + 2*(7*sin(2*x) - 2*sin(x))*sin(3*x) - 2*sin(3*x)^2 - 2
4*sin(2*x)^2 + 14*sin(2*x)*sin(x) - 2*sin(x)^2 + cos(x))/(2*(2*cos(3*x) - 6
*cos(2*x) + 2*cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 4*(6*cos(2*x) - 2*cos(x)
+ 1)*cos(3*x) - 4*cos(3*x)^2 + 12*(2*cos(x) - 1)*cos(2*x) - 36*cos(2*x)^2 -
 4*cos(x)^2 + 4*(sin(3*x) - 3*sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 8*
(3*sin(2*x) - sin(x))*sin(3*x) - 4*sin(3*x)^2 - 36*sin(2*x)^2 + 24*sin(2*x)
*sin(x) - 4*sin(x)^2 + 4*cos(x) - 1), x) - 2*sin(2*x))/(cos(2*x)^2 + sin(2*
x)^2 - 2*cos(2*x) + 1)
```

mupad [B] time = 2.29, size = 95, normalized size = 1.34

$$-\frac{1}{3\tan(x)}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\,\sqrt{6}\,\tan(x)\left(\frac{1}{27}-\frac{1}{27}\mathrm{i}\right)}{-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\left(3^{1/4}\,(1+1\mathrm{i})+3^{3/4}\,(-1+1\mathrm{i})\right)1\mathrm{i}}{36}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\,\sqrt{6}\,\tan(x)\left(\frac{1}{27}+\frac{1}{27}\mathrm{i}\right)}{\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\left(3^{1/}\right.}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^6 - 1),x)
```

```
[Out] (6^(1/2)*atan((3^(1/4)*6^(1/2)*tan(x)*(1/27 - 1i/27))/((3^(1/2)*1i)/9 - 1/9
))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 - 1/(3*tan(x)) + (6^(1/2)*a
tan((3^(1/4)*6^(1/2)*tan(x)*(1/27 + 1i/27))/((3^(1/2)*1i)/9 + 1/9))*(3^(1/4
)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

sympy [B] time = 23.61, size = 728, normalized size = 10.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**6),x)
```

```
[Out] sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*floor((x/2 - pi/
2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*flo
or((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2)
+ 1) + pi*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(1
/4)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(
sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*
3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi/2)/pi)
)/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 + 1) + pi*floor((
x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 +
1) + pi*floor((x/2 - pi/2)/pi))/12 - sqrt(2)*3**(1/4)*log(4*tan(x/2)**2 - 4
*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/24 + sqrt(2)*3**(3/4)*log(4*tan(x/2
)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 - sqrt(2)*3**(3/4)*log(4
*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 + sqrt(2)*3**(1/
4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/24 - sqrt(2
)*3**(3/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/
72 + sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2) + 1
2*sqrt(3))/24 - sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 + 12*sqrt(2)*3**(3/4)*t
an(x/2) + 12*sqrt(3))/24 + sqrt(2)*3**(3/4)*log(36*tan(x/2)**2 + 12*sqrt(2)
*3**(3/4)*tan(x/2) + 12*sqrt(3))/72 + tan(x/2)/6 - 1/(6*tan(x/2))
```

3.85 $\int \frac{1}{1-\cos^8(x)}\,dx$

Optimal. Leaf size=89

$$\frac{x}{4\sqrt{2}}-\frac{\cot(x)}{4}-\frac{\tan^{-1}\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}}-\frac{\tan^{-1}\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] -1/4*cot(x)-1/4*arctan(cot(x)*(1-I)^(1/2))/(1-I)^(1/2)-1/4*arctan(cot(x)*(1
+I)^(1/2))/(1+I)^(1/2)+1/8*x*2^(1/2)-1/8*arctan(cos(x)*sin(x)/(1+cos(x)^2+2
^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 203, 3175, 3767, 8}

$$\frac{x}{4\sqrt{2}}-\frac{\cot(x)}{4}-\frac{\tan^{-1}\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}}-\frac{\tan^{-1}\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^8)^(-1),x]
```

```
[Out] x/(4*Sqrt[2]) - ArcTan[Sqrt[1 - I]*Cot[x]]/(4*Sqrt[1 - I]) - ArcTan[Sqrt[1
+ I]*Cot[x]]/(4*Sqrt[1 + I]) - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]
^2)]/(4*Sqrt[2]) - Cot[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-i\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+i\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\cos^2(x)}\,dx$$
$$= \frac{1}{4}\int \csc^2(x)\,dx - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1-i)x^2}\,dx, x, \cot(x)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1+i)x^2}\,dx, x$$
$$= \frac{x}{4\sqrt{2}} - \frac{\tan^{-1}\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}} - \frac{\tan^{-1}\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}} - \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}} - \frac{1}{4}\text{Subst}($$
$$= \frac{x}{4\sqrt{2}} - \frac{\tan^{-1}\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}} - \frac{\tan^{-1}\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}} - \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}} - \frac{\cot(x)}{4}$$

Mathematica [A] time = 0.16, size = 64, normalized size = 0.72

$$\frac{1}{8}\left(\frac{2\tan^{-1}\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{2\tan^{-1}\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + \sqrt{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) - 2\cot(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^8)^(-1),x]
[Out] ((2*ArcTan[Tan[x]/Sqrt[1 - I]])/Sqrt[1 - I] + (2*ArcTan[Tan[x]/Sqrt[1 + I]]
)/Sqrt[1 + I] + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/8
```

fricas [B] time = 63.66, size = 3963, normalized size = 44.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^8),x, algorithm="fricas")
[Out] -1/64*(2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(1/4*(32*(s
qrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2)
 + 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(
2) - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(
x)^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2
)*(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1)
 - 2*sqrt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*s
qrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2)
+ 5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58)
)*cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos
(x)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7
 + 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2
^(3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))
*sqrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8
*sqrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(
x)^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqr
t(2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2)
- 46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2)
 + 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*
sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(
2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2)
```

```
- 39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23)
- 2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^
(1/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(s
qrt(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*s
in(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 + 4
*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*s
qrt(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 2
56*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)
) - 2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqr
t(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) +
 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2)
 - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)
^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*
(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) -
 2*sqrt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqr
t(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) +
5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*
cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x
)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 +
 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(
3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*s
qrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*s
qrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)
^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(
2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) -
46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) +
 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sq
rt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2)
 - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) -
39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) -
2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1
/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqr
t(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin
(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 + 4*(
2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqr
t(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256
*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))
+ 2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(
2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 2
2) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) -
 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^1
0 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(4
9*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) - 2
*sqrt(2) - 5)*cos(x)^4 - 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(
2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) + 5)
)*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*co
s(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x)^
9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 + 2
*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(3/
4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*sqr
t(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqr
t(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^1
2 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)
*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46
) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 7
2)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^(3/4)*(8*sqrt
(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) -
 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39
) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^
```

```
(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4
)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(
2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x
) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 - 4*(2^
(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(
2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*c
os(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) -
2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(1/4*(32*(sqrt(2)*
(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 22)
- 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) - 37
)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^10 -
 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(49*s
qrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) - 2*sq
rt(2) - 5)*cos(x)^4 - 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2)
+ 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) + 5))*c
os(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*cos(x
)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x)^9 -
 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 + 2*(2
^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(3/4)*
(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*sqrt(2
*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2
) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 +
 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7
*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) -
 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*
cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^(3/4)*(8*sqrt(2)
 - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11
) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) -
 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/
4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(
4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2)
- 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) -
 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 - 4*(2^(1/
4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2)
+ 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(
x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) + (2^
(1/4)*(sqrt(2) - 1)*cos(x)^2 - 2^(1/4)*(sqrt(2) - 1))*sqrt(2*sqrt(2) + 4)*l
og(-(4*sqrt(2) - 5)*cos(x)^4 + 4*(sqrt(2) - 1)*cos(x)^2 + (2^(1/4)*(3*sqrt(
2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x)
 + 1) - (2^(1/4)*(sqrt(2) - 1)*cos(x)^2 - 2^(1/4)*(sqrt(2) - 1))*sqrt(2*sqr
t(2) + 4)*log(-(4*sqrt(2) - 5)*cos(x)^4 + 4*(sqrt(2) - 1)*cos(x)^2 - (2^(1/
4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2)
+ 4)*sin(x) + 1) + 4*(sqrt(2)*cos(x)^2 - sqrt(2))*arctan(1/4*(3*sqrt(2)*cos
(x)^2 - sqrt(2))/(cos(x)*sin(x))) - 16*cos(x)*sin(x))/(cos(x)^2 - 1)
```

giac [B] time = 0.88, size = 222, normalized size = 2.49

$$\frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan\right.}{2\sqrt{\sqrt{2}+2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^8),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4)*
(2^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) +
1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2)
```

```
 + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) - 1/16*sqrt(sqrt(2)
 - 1)*log(tan(x)^2 + 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1/16*sq
rt(sqrt(2) - 1)*log(tan(x)^2 - 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2))
 - 1/4/tan(x)
```

maple [B] time = 0.11, size = 246, normalized size = 2.76

$$\frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{8}+\frac{\sqrt{2}\,\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{32}+\frac{\arctan\left(\frac{2\tan(x)-\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right)\sqrt{2}}{8\sqrt{2\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^8),x)
```

```
[Out] 1/8*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)+1/32*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln
(tan(x)^2-tan(x)*(-2+2*2^(1/2))^(1/2)+2^(1/2))+1/8/(2*2^(1/2)+2)^(1/2)*arct
an((2*tan(x)-(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2))*2^(1/2)+1/4/(2*2^(1
/2)+2)^(1/2)*arctan((2*tan(x)-(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2))-1/
32*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(tan(x)^2+tan(x)*(-2+2*2^(1/2))^(1/2)+2^(
1/2))+1/8/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)+(-2+2*2^(1/2))^(1/2))/(2*2^(
1/2)+2)^(1/2))*2^(1/2)+1/4/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)+(-2+2*2^(1/
2))^(1/2))/(2*2^(1/2)+2)^(1/2))-1/4/tan(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2}\left(\sqrt{2}\cos(2x)^2+\sqrt{2}\sin(2x)^2-2\sqrt{2}\cos(2x)+\sqrt{2}\right)\left(2\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\right)\sqrt{\sqrt{2}+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^8),x, algorithm="maxima")
```

```
[Out] 1/8*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*arctan2(4*sqrt(2)*sin(2*x)/
(2*(2*sqrt(2) + 3)*cos(2*x) + cos(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17), (
cos(2*x)^2 + sin(2*x)^2 + 6*cos(2*x) + 1)/(2*(2*sqrt(2) + 3)*cos(2*x) + cos
(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17)) + 64*(sqrt(2)*cos(2*x)^2 + sqrt(2)
*sin(2*x)^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))*integrate(((4*cos(2*x) + 1)*cos
(4*x) + cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) + 22*cos(4*x)^2 + sin(8*x)*
sin(4*x) + 4*sin(6*x)*sin(4*x) + 22*sin(4*x)^2 + 4*sin(4*x)*sin(2*x))/(2*(4
*cos(6*x) + 22*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(22*cos
(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 44*(4*cos(2*x) + 1)*cos(
4*x) + 484*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 11*sin(4*x) + 2*sin
(2*x))*sin(8*x) + sin(8*x)^2 + 16*(11*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*
sin(6*x)^2 + 484*sin(4*x)^2 + 176*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos
(2*x) + 1), x) - 4*sqrt(2)*sin(2*x))/(sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)
^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))
```

mupad [B] time = 2.27, size = 241, normalized size = 2.71

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8}-\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}\,1i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)}+\frac{\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}\,1i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)}\right)\left(\sqrt{\vphantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^8 - 1),x)
```

```
[Out] atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256
- 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) + 1/16)) - (2^(1/2)*tan(x)*(2^
(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/2)/
256 - 1/256)^(1/2) + 1/16)))*((- 2^(1/2)/256 - 1/256)^(1/2)*2i + (2^(1/2)/2
56 - 1/256)^(1/2)*2i) - atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*
1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) - 1/16
)) + (2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256 -
1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) - 1/16)))*((- 2^(1/2)/256 - 1/25
6)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) - 1/(4*tan(x)) + (2^(1/2)*ata
n((2^(1/2)*tan(x))/2))/8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**8),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\tan(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(\cos^2(x)+1\right)-\log(\cos(x))$$

```
[Out] -ln(cos(x))+1/2*ln(1+cos(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3194, 36, 29, 31}

$$\frac{1}{2}\log\left(\cos^2(x)+1\right)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(1 + Cos[x]^2),x]
```

```
[Out] -Log[Cos[x]] + Log[1 + Cos[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{1+\cos^2(x)}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,\cos^2(x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,\cos^2(x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\cos^2(x)\right)\\ &= -\log(\cos(x))+\frac{1}{2}\log\left(1+\cos^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{2}\log\left(\cos^2(x)+1\right)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(1 + Cos[x]^2),x]
[Out] -Log[Cos[x]] + Log[1 + Cos[x]^2]/2
```

fricas [A] time = 0.53, size = 19, normalized size = 1.12

$$\frac{1}{2}\log\left(\frac{1}{2}\cos(x)^2 + \frac{1}{2}\right) - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2),x, algorithm="fricas")
[Out] 1/2*log(1/2*cos(x)^2 + 1/2) - log(-cos(x))
```

giac [A] time = 0.45, size = 16, normalized size = 0.94

$$\frac{1}{2}\log\left(\cos(x)^2 + 1\right) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2),x, algorithm="giac")
[Out] 1/2*log(cos(x)^2 + 1) - log(abs(cos(x)))
```

maple [A] time = 0.10, size = 16, normalized size = 0.94

$$-\ln(\cos(x)) + \frac{\ln\left(1+\cos^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+cos(x)^2),x)
[Out] -ln(cos(x))+1/2*ln(1+cos(x)^2)
```

maxima [A] time = 0.30, size = 19, normalized size = 1.12

$$-\frac{1}{2}\log\left(\sin(x)^2 - 1\right) + \frac{1}{2}\log\left(\sin(x)^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2),x, algorithm="maxima")
[Out] -1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 2)
```

mupad [B] time = 2.16, size = 9, normalized size = 0.53

$$\frac{\ln\left(\tan(x)^2 + 2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)^2 + 1),x)
[Out] log(tan(x)^2 + 2)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\cos^2(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)**2),x)
```

```
[Out] Integral(tan(x)/(cos(x)**2 + 1), x)
```

3.87 $\int \sqrt{a+b\cos^2(x)}\,\tan(x)\,dx$

Optimal. Leaf size=40

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\cos^2(x)}$$

```
[Out] arctanh((a+b*cos(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*cos(x)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3194, 50, 63, 208}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^2]*Tan[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]] - Sqrt[a + b*Cos[x]^2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\cos^2(x)}\,\tan(x)\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\sqrt{a+b\cos^2(x)} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^2(x)\right)$$
$$= -\sqrt{a+b\cos^2(x)} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^2(x)}\right)}{b}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\cos^2(x)}$$

Mathematica [**A**] time = 0.03, size = 40, normalized size = 1.00

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^2]*Tan[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]] - Sqrt[a + b*Cos[x]^2]
```

fricas [**A**] time = 2.08, size = 90, normalized size = 2.25

$$\left[\frac{1}{2}\sqrt{a}\log\left(\frac{b\cos(x)^2 + 2\sqrt{b\cos(x)^2+a}\sqrt{a} + 2a}{\cos(x)^2}\right) - \sqrt{b\cos(x)^2+a}, -\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^2+a}\sqrt{-a}}{a}\right) - \sqrt{b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((b*cos(x)^2 + 2*sqrt(b*cos(x)^2 + a)*sqrt(a) + 2*a)/cos(x)
^2) - sqrt(b*cos(x)^2 + a), -sqrt(-a)*arctan(sqrt(b*cos(x)^2 + a)*sqrt(-a)/
a) - sqrt(b*cos(x)^2 + a)]
```

giac [**A**] time = 0.40, size = 38, normalized size = 0.95

$$-\frac{a\arctan\left(\frac{\sqrt{b\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \sqrt{b\cos(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -a*arctan(sqrt(b*cos(x)^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*cos(x)^2 + a)
```

maple [**A**] time = 0.08, size = 43, normalized size = 1.08

$$-\sqrt{a+b\left(\cos^2(x)\right)} + \sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^2(x)\right)}}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)^(1/2)*tan(x),x)
```

```
[Out] -(a+b*cos(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^2)^(1/2))/cos(x
))
```

maxima [B] time = 1.14, size = 95, normalized size = 2.38

$$\frac{1}{2}\sqrt{a}\log\left(b-\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)-1}-\frac{a}{\sin(x)-1}\right)+\frac{1}{2}\sqrt{a}\log\left(-b+\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)+1}+\frac{a}{\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*log(b - sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) - 1) - a/(sin
(x) - 1)) + 1/2*sqrt(a)*log(-b + sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x)
+ 1) + a/(sin(x) + 1)) - sqrt(-b*sin(x)^2 + a + b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\sqrt{b\cos(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**2)*tan(x), x)
```

3.88 $\int \sqrt{1-\cos^2(x)} \tan(x) \, dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)-\sqrt{\sin^2(x)}$$

```
[Out] arctanh((sin(x)^2)^(1/2))-(sin(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3176, 3205, 50, 63, 206}

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)-\sqrt{\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]^2]*Tan[x],x]
```

```
[Out] ArcTanh[Sqrt[Sin[x]^2]] - Sqrt[Sin[x]^2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\cos^2(x)}\,\tan(x)\,dx &= \int \sqrt{\sin^2(x)}\,\tan(x)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{x}}{1-x}\,dx, x, \sin^2(x)\right)\\ &= -\sqrt{\sin^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}}\,dx, x, \sin^2(x)\right)\\ &= -\sqrt{\sin^2(x)} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin^2(x)}\right)\\ &= \tanh^{-1}\left(\sqrt{\sin^2(x)}\right) - \sqrt{\sin^2(x)}\end{aligned}$$

Mathematica [B] time = 0.04, size = 47, normalized size = 2.35

$$\sqrt{\sin^2(x)}\,(-\csc(x))\left(\sin(x)+\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]^2]*Tan[x],x]
[Out] -(Csc[x]*Sqrt[Sin[x]^2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]
] + Sin[x]))
```

fricas [A] time = 0.50, size = 21, normalized size = 1.05

$$\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(-\sin(x)+1\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

giac [B] time = 0.37, size = 45, normalized size = 2.25

$$-\sqrt{-\cos(x)^2+1}+\frac{1}{2}\log\left(\sqrt{-\cos(x)^2+1}+1\right)-\frac{1}{2}\log\left(-\sqrt{-\cos(x)^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="giac")
[Out] -sqrt(-cos(x)^2 + 1) + 1/2*log(sqrt(-cos(x)^2 + 1) + 1) - 1/2*log(-sqrt(-co
s(x)^2 + 1) + 1)
```

maple [A] time = 0.99, size = 17, normalized size = 0.85

$$-\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}+\operatorname{arctanh}\left(\frac{2}{\sqrt{2-2\cos(2x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)^(1/2)*tan(x),x)
[Out] -(sin(x)^2)^(1/2)+arctanh(1/(sin(x)^2)^(1/2))
```

maxima [B] time = 0.96, size = 47, normalized size = 2.35

$$\frac{1}{2}\,(-1)^{2\,\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)+1}\right)+\frac{1}{2}\,(-1)^{2\,\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)-1}\right)-\sqrt{\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*sin(x))*log(-sin(x)/(sin(x) + 1)) + 1/2*(-1)^(2*sin(x))*log(-si
n(x)/(sin(x) - 1)) - sqrt(sin(x)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(x)\,\sqrt{1-\cos(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1 - cos(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)*(1 - cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(\cos\left(x\right)-1\right)\left(\cos\left(x\right)+1\right)}\,\tan\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(-(cos(x) - 1)*(cos(x) + 1))*tan(x), x)
```

3.89 $\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] arctanh((a+b*cos(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]]/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^2(x)\right)\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]]/Sqrt[a]
```

fricas [A] time = 0.80, size = 67, normalized size = 2.68

$$\left[\frac{\log\left(\frac{b\cos(x)^2+2\sqrt{b\cos(x)^2+a}\sqrt{a}+2a}{\cos(x)^2}\right)}{2\sqrt{a}}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^2+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cos(x)^2 + 2*sqrt(b*cos(x)^2 + a)*sqrt(a) + 2*a)/cos(x)^2)/sqrt
(a), -sqrt(-a)*arctan(sqrt(b*cos(x)^2 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.31, size = 24, normalized size = 0.96

$$-\frac{\arctan\left(\frac{\sqrt{b\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b*cos(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.10, size = 30, normalized size = 1.20

$$\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\cos^2(x)\right)}}{\cos(x)}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^2)^(1/2),x)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^2)^(1/2))/cos(x))
```

maxima [B] time = 0.89, size = 81, normalized size = 3.24

$$\frac{\log\left(b-\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)-1}-\frac{a}{\sin(x)-1}\right)}{2\sqrt{a}}+\frac{\log\left(-b+\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)+1}+\frac{a}{\sin(x)+1}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b - sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) - 1) - a/(sin(x) - 1)
)/sqrt(a) + 1/2*log(-b + sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) + 1) + a
/(sin(x) + 1))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{b\cos(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^2)^(1/2),x)
[Out] int(tan(x)/(a + b*cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{(x)}}{\sqrt{a + b\cos^2{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**2)**(1/2),x)
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**2), x)
```

3.90 $\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=11

$$\tanh^{-1}\left(\sqrt{\cos^2(x)+1}\right)$$

```
[Out] arctanh((1+cos(x)^2)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3194, 63, 207}

$$\tanh^{-1}\left(\sqrt{\cos^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[1 + Cos[x]^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \cos^2(x)\right)\right)\\
&= -\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\cos^2(x)}\right)\\
&= \tanh^{-1}\left(\sqrt{1+\cos^2(x)}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\tanh^{-1}\left(\sqrt{\cos^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[1 + Cos[x]^2],x]
[Out] ArcTanh[Sqrt[1 + Cos[x]^2]]
```

fricas [A] time = 0.67, size = 16, normalized size = 1.45

$$\log\left(\frac{\sqrt{\cos(x)^2+1}+1}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] log((sqrt(cos(x)^2 + 1) + 1)/cos(x))
```

giac [B] time = 0.37, size = 27, normalized size = 2.45

$$\frac{1}{2}\log\left(\sqrt{\cos(x)^2+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\cos(x)^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(sqrt(cos(x)^2 + 1) + 1) - 1/2*log(sqrt(cos(x)^2 + 1) - 1)
```

maple [A] time = 0.09, size = 10, normalized size = 0.91

$$\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\cos^2(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+cos(x)^2)^(1/2),x)
[Out] arctanh(1/(1+cos(x)^2)^(1/2))
```

maxima [B] time = 0.88, size = 60, normalized size = 5.45

$$\frac{1}{2}\log\left(\frac{\sqrt{-\sin(x)^2+2}}{\sin(x)+1}+\frac{1}{\sin(x)+1}-1\right)+\frac{1}{2}\log\left(-\frac{\sqrt{-\sin(x)^2+2}}{\sin(x)-1}-\frac{1}{\sin(x)-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(sqrt(-sin(x)^2 + 2)/(sin(x) + 1) + 1/(sin(x) + 1) - 1) + 1/2*log(-s
qrt(-sin(x)^2 + 2)/(sin(x) - 1) - 1/(sin(x) - 1) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\tan(x)}{\sqrt{\cos(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)^2 + 1)^(1/2),x)
[Out] int(tan(x)/(cos(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(cos(x)**2 + 1), x)
```

3.91 $\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}} \, dx$

Optimal. Leaf size=9

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)$$

[Out] arctanh((sin(x)^2)^(1/2))

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3176, 3205, 63, 206}

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/Sqrt[1 - Cos[x]^2],x]

[Out] ArcTanh[Sqrt[Sin[x]^2]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx &= \int \frac{\tan(x)}{\sqrt{\sin^2(x)}}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}}\,dx, x, \sin^2(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin^2(x)}\right) \\
&= \tanh^{-1}\left(\sqrt{\sin^2(x)}\right)
\end{aligned}
$$

Mathematica [B] time = 0.02, size = 44, normalized size = 4.89

$$\frac{\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[1 - Cos[x]^2],x]
[Out] ((-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sin[x])/Sqrt[Sin[x]
^2]
```

fricas [B] time = 0.64, size = 17, normalized size = 1.89

$$\frac{1}{2}\log\left(\sin(x)+1\right)-\frac{1}{2}\log\left(-\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

giac [B] time = 0.34, size = 33, normalized size = 3.67

$$\frac{1}{2}\log\left(\sqrt{-\cos(x)^2+1}+1\right)-\frac{1}{2}\log\left(-\sqrt{-\cos(x)^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(sqrt(-cos(x)^2 + 1) + 1) - 1/2*log(-sqrt(-cos(x)^2 + 1) + 1)
```

maple [A] time = 0.44, size = 8, normalized size = 0.89

$$\operatorname{arctanh}\left(\frac{2}{\sqrt{2-2\cos(2x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1-cos(x)^2)^(1/2),x)
[Out] arctanh(1/(sin(x)^2)^(1/2))
```

maxima [B] time = 0.58, size = 39, normalized size = 4.33

$$\frac{1}{2}\,(-1)^{2\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)+1}\right)+\frac{1}{2}\,(-1)^{2\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*sin(x))*log(-sin(x)/(sin(x) + 1)) + 1/2*(-1)^(2*sin(x))*log(-si
n(x)/(sin(x) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\tan(x)}{\sqrt{1-\cos(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1 - cos(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)/(1 - cos(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{(x)}}{\sqrt{-(\cos{(x)}-1)(\cos{(x)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(-(cos(x) - 1)*(cos(x) + 1)), x)
```

3.92 $\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx$

Optimal. Leaf size=153

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\cos^2(x)\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cos(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\log\left(a+\right.$$

```
[Out] ln(cos(x))/a+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*cos(x))/a^(5/3)-1/6*b^(2/3)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*cos(x)+b^(2/3)*cos(x)^2)/a^(5/3)-1/3*ln(a+b*cos(x)^
3)/a+1/2*sec(x)^2/a-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*cos(x))/a^(1/
3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3230, 1834, 1871, 200, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\cos^2(x)\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cos(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\log\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Cos[x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Cos[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(5/3))) + Log[Cos[x]]/a + (b^(2/3)*Log[a^(1/3) + b^(1/3)*Cos[x]])/(3*a^(
5/3)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Cos[x] + b^(2/3)*Cos[x]^2])/
(6*a^(5/3)) - Log[a + b*Cos[x]^3]/(3*a) + Sec[x]^2/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = -\text{Subst}\left(\int \frac{1-x^2}{x^3\left(a+bx^3\right)}\,dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{ax}+\frac{b\left(-1+x^2\right)}{a\left(a+bx^3\right)}\right)dx, x, \cos(x)\right)$$

$$= \frac{\log(\cos(x))}{a}+\frac{\sec^2(x)}{2a}-\frac{b\,\text{Subst}\left(\int \frac{-1+x^2}{a+bx^3}\,dx, x, \cos(x)\right)}{a}$$

$$= \frac{\log(\cos(x))}{a}+\frac{\sec^2(x)}{2a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \cos(x)\right)}{a}-\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \cos(x)\right)}{a}$$

$$= \frac{\log(\cos(x))}{a}-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \cos(x)\right)}{3a^{5/3}}+\frac{b\,\text{Subs}}{}$$

$$= \frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}-\frac{b^{2/3}\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\cos^2(x)\right)}{6a^{5/3}}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\cos(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.30, size = 217, normalized size = 1.42

$$\frac{-2\text{RootSum}\left[\#1^3a-\#1^3b+3\#1^2a+3\#1^2b+3\#1a-3\#1b+a+b\&, \frac{\#1^2a\log\left(\tan^2\left(\frac{x}{2}\right)-\#1\right)-\#1^2b\log\left(\tan^2\left(\frac{x}{2}\right)-\#1\right)+2\#1a\log}{\#1^2}\right.}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Cos[x]^3),x]
```

```
[Out] (6*(Log[Cos[x]] + Log[Sec[x/2]^2]) - 2*RootSum[a + b + 3*a*#1 - 3*b*#1 + 3*
a*#1^2 + 3*b*#1^2 + a*#1^3 - b*#1^3 & , (a*Log[-#1 + Tan[x/2]^2] + b*Log[-#
1 + Tan[x/2]^2] + 2*a*Log[-#1 + Tan[x/2]^2]*#1 + 4*b*Log[-#1 + Tan[x/2]^2]*
#1 + a*Log[-#1 + Tan[x/2]^2]*#1^2 - b*Log[-#1 + Tan[x/2]^2]*#1^2)/(a - b +
2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] + 3*Sec[x]^2)/(6*a)
```

fricas [C] time = 46.28, size = 1690, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(1/3)*a*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (
a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2)*arctan(-1/8*(2*sqrt(1
/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(
1/3) + 2/a)^2*a^4 + 4*b^2*cos(x)^2 - 4*a*b*cos(x) + 2*(a^2*b*cos(x) - 2*a^3
)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
2/a) + 4*a^2)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/
```

```
a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2) + sqrt(1/3
)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
 2/a)^2*a^5 - 8*a^2*b*cos(x) + 4*a^3 + 4*(a^3*b*cos(x) - a^4)*((1/2)^(1/3)*
(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a))*sqrt((((1
/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^
2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^
(1/3) + 2/a)*a + 4)/a^2))/b^2)*cos(x)^2 - 6*sqrt(1/3)*a*sqrt((((1/2)^(1/3)*
(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*
((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/
a)*a + 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^4 + 4*b^2*cos(x)^2 - 4*
a*b*cos(x) + 2*(a^2*b*cos(x) - 2*a^3)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
 b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a) + 4*a^2)*(((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((
1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)
^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)
^(1/3) + 2/a)*a + 4)/a^2) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^5 - 8*a^2*b*cos(x) + 4*a^3 +
4*(a^3*b*cos(x) - a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2
 - b^2)/a^5)^(1/3) + 2/a))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/
a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2))/b^2)*cos(x)^2
 - ((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
 2/a)*a*cos(x)^2*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a
^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^4 + b^2*cos(x)^2 + 2*a*b*cos(x) - (a^2*b*co
s(x) + a^3)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5
)^(1/3) + 2/a) + a^2) + 12*cos(x)^2*log(-cos(x)) + (((1/2)^(1/3)*(I*sqrt(3)
 + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a*cos(x)^2 - 6*cos(x
)^2)*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(
1/3) + 2/a)^2*a^4 + 4*b^2*cos(x)^2 - 4*a*b*cos(x) + 2*(a^2*b*cos(x) - 2*a^3
)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
2/a) + 4*a^2) + 6)/(a*cos(x)^2)
```

giac [A] time = 2.13, size = 143, normalized size = 0.93

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\cos(x)\right|\right)}{3\,a^2}-\frac{\log\left(\left|b\cos(x)^3+a\right|\right)}{3\,a}+\frac{\log\left(\left|\cos(x)\right|\right)}{a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\cos(\cdots\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + cos(x)))/a^2 - 1/3*log(abs(b*co
s(x)^3 + a))/a + log(abs(cos(x)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3
*sqrt(3)*((-a/b)^(1/3) + 2*cos(x))/(-a/b)^(1/3))/a^2 + 1/6*(-a*b^2)^(1/3)*l
og(cos(x)^2 + (-a/b)^(1/3)*cos(x) + (-a/b)^(2/3))/a^2 + 1/2/(a*cos(x)^2)
```

maple [A] time = 0.11, size = 125, normalized size = 0.82

$$\frac{\ln\left(\cos(x)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos^2(x)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(x)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(a+b\left(\cos^3(x)\right)\right)}{3a}+\frac{\ln\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*cos(x)^3),x)
```

```
[Out] 1/3/a/(1/b*a)^(2/3)*ln(cos(x)+(1/b*a)^(1/3))-1/6/a/(1/b*a)^(2/3)*ln(cos(x)^
2-(1/b*a)^(1/3)*cos(x)+(1/b*a)^(2/3))+1/3/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*cos(x)-1))-1/3*ln(a+b*cos(x)^3)/a+ln(cos(x))/a+1
/2/a/cos(x)^2
```

maxima [A] time = 1.59, size = 151, normalized size = 0.99

$$\frac{\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\cos(x)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2}-\frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(x)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b)^(1
/3) - 2*cos(x))/(a/b)^(1/3))/a^2 - 1/6*(2*(a/b)^(2/3) + 1)*log(cos(x)^2 - (
a/b)^(1/3)*cos(x) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*((a/b)^(2/3) - 1)*lo
g((a/b)^(1/3) + cos(x))/(a*(a/b)^(2/3)) + log(cos(x))/a + 1/2/(a*cos(x)^2)
```

mupad [B] time = 5.21, size = 1281, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*cos(x)^3),x)
```

```
[Out] (2*tan(x/2)^2)/(a - 2*a*tan(x/2)^2 + a*tan(x/2)^4) + log(tan(x/2)^2 - 1)/a
+ symsum(log((262144*(9*a*b^10 - b^11 - 37*a^2*b^9 + 85*a^3*b^8 - 107*a^4*b
^7 + 43*a^5*b^6 + 73*a^6*b^5 - 121*a^7*b^4 + 72*a^8*b^3 - 16*a^9*b^2))/a^6
+ root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(27*a^5*z^
3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(27*a^5*z^3 + 27*a^4*z^2 +
 9*a^3*z + a^2 - b^2, z, k)*((262144*(72*a^5*b^9 - 96*a^6*b^8 + 1428*a^7*b^
7 - 3684*a^8*b^6 + 612*a^9*b^5 + 3972*a^10*b^4 - 2112*a^11*b^3 - 192*a^12*b
^2))/a^6 + root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(
27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*((262144*(5184*a^10*b^
6 - 3024*a^9*b^7 + 1728*a^11*b^5 - 6048*a^12*b^4 + 1296*a^13*b^3 + 864*a^14
*b^2))/a^6 - root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*((26
2144*(1296*a^10*b^7 - 3888*a^11*b^6 + 2592*a^12*b^5 + 2592*a^13*b^4 - 3888*
a^14*b^3 + 1296*a^15*b^2))/a^6 - (262144*tan(x/2)^2*(1296*a^10*b^7 - 11016*
a^11*b^6 + 27216*a^12*b^5 - 28512*a^13*b^4 + 12960*a^14*b^3 - 1944*a^15*b^2
))/a^6) + (262144*tan(x/2)^2*(4104*a^9*b^7 - 16740*a^10*b^6 + 18468*a^11*b^
5 - 1836*a^12*b^4 - 5292*a^13*b^3 + 1296*a^14*b^2))/a^6) + (262144*(288*a^7
*b^8 - 1836*a^8*b^7 - 1692*a^9*b^6 + 6084*a^10*b^5 + 108*a^11*b^4 - 4248*a^
12*b^3 + 1296*a^13*b^2))/a^6 + (262144*tan(x/2)^2*(4392*a^8*b^7 - 360*a^7*b
^8 + 3366*a^9*b^6 - 29934*a^10*b^5 + 35946*a^11*b^4 - 15354*a^12*b^3 + 1944
*a^13*b^2))/a^6) - (262144*tan(x/2)^2*(72*a^5*b^9 - 162*a^6*b^8 + 3780*a^7*
b^7 - 20160*a^8*b^6 + 30276*a^9*b^5 - 14526*a^10*b^4 + 432*a^11*b^3 + 288*a
^12*b^2))/a^6) + (262144*(68*a^4*b^9 - 436*a^5*b^8 + 903*a^6*b^7 - 55*a^7*b
^6 - 1579*a^8*b^5 + 987*a^9*b^4 + 608*a^10*b^3 - 496*a^11*b^2))/a^6 - (2621
44*tan(x/2)^2*(90*a^4*b^9 - 666*a^5*b^8 + 3753*a^6*b^7 - 5925*a^7*b^6 - 131
1*a^8*b^5 + 8919*a^9*b^4 - 5604*a^10*b^3 + 744*a^11*b^2))/a^6) - (262144*(8
*a^2*b^10 - 26*a^3*b^9 - 30*a^4*b^8 + 292*a^5*b^7 - 540*a^6*b^6 + 230*a^7*b
^5 + 402*a^8*b^4 - 496*a^9*b^3 + 160*a^10*b^2))/a^6 + (262144*tan(x/2)^2*(1
```

```
0*a^2*b^10 - 54*a^3*b^9 + 52*a^4*b^8 + 920*a^5*b^7 - 4214*a^6*b^6 + 7442*a^
7*b^5 - 6168*a^8*b^4 + 2252*a^9*b^3 - 240*a^10*b^2))/a^6) - (262144*tan(x/2
)^2*(11*a*b^10 - b^11 - 87*a^2*b^9 + 391*a^3*b^8 - 1045*a^4*b^7 + 1705*a^5*
b^6 - 1677*a^6*b^5 + 941*a^7*b^4 - 262*a^8*b^3 + 24*a^9*b^2))/a^6)*root(27*
a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k), k, 1, 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{a + b\cos^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*cos(x)**3),x)
```

```
[Out] Integral(tan(x)**3/(a + b*cos(x)**3), x)
```

3.93 $\int \sqrt{a+b\cos^3(x)}\,\tan(x)\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+b\cos^3(x)}$$

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))*a^(1/2)-2/3*(a+b*cos(x)^3)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+b\cos^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^3]*Tan[x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/3 - (2*Sqrt[a + b*Cos[x]^
3])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^3(x)\right)\right)$$
$$= -\frac{2}{3}\sqrt{a+b\cos^3(x)} - \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^3(x)\right)$$
$$= -\frac{2}{3}\sqrt{a+b\cos^3(x)} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^3(x)}\right)}{3b}$$
$$= \frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right) - \frac{2}{3}\sqrt{a+b\cos^3(x)}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right) - \frac{2}{3}\sqrt{a+b\cos^3(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^3]*Tan[x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/3 - (2*Sqrt[a + b*Cos[x]^
3])/3
```

fricas [A] time = 5.09, size = 123, normalized size = 2.73

$$\left[\frac{1}{6}\sqrt{a}\log\left(-\frac{b^2\cos(x)^6 + 8\,ab\cos(x)^3 + 4\left(b\cos(x)^3 + 2\,a\right)\sqrt{b\cos(x)^3 + a}\,\sqrt{a} + 8\,a^2}{\cos(x)^6}\right) - \frac{2}{3}\sqrt{b\cos(x)^3 + a}, -\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-(b^2*cos(x)^6 + 8*a*b*cos(x)^3 + 4*(b*cos(x)^3 + 2*a)*sqr
t(b*cos(x)^3 + a)*sqrt(a) + 8*a^2)/cos(x)^6) - 2/3*sqrt(b*cos(x)^3 + a), -1
/3*sqrt(-a)*arctan(2*sqrt(b*cos(x)^3 + a)*sqrt(-a)/(b*cos(x)^3 + 2*a)) - 2/
3*sqrt(b*cos(x)^3 + a)]
```

giac [A] time = 0.51, size = 38, normalized size = 0.84

$$-\frac{2\,a\arctan\left(\frac{\sqrt{b\cos(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} - \frac{2}{3}\sqrt{b\cos(x)^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -2/3*a*arctan(sqrt(b*cos(x)^3 + a)/sqrt(-a))/sqrt(-a) - 2/3*sqrt(b*cos(x)^3
 + a)
```

maple [A] time = 0.70, size = 34, normalized size = 0.76

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cos^{3}(x)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}-\frac{2\sqrt{a+b\left(\cos^{3}(x)\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^3)^(1/2)*tan(x),x)
```

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))*a^(1/2)-2/3*(a+b*cos(x)^3)^(1/2)
```

maxima [A] time = 0.77, size = 52, normalized size = 1.16

$$-\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{b\cos(x)^{3}+a}-\sqrt{a}}{\sqrt{b\cos(x)^{3}+a}+\sqrt{a}}\right)-\frac{2}{3}\sqrt{b\cos(x)^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(a)*log((sqrt(b*cos(x)^3 + a) - sqrt(a))/(sqrt(b*cos(x)^3 + a) + s
qrt(a))) - 2/3*sqrt(b*cos(x)^3 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\sqrt{b\cos(x)^{3}+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^3)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos^{3}(x)}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**3)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**3)*tan(x), x)
```

3.94 $\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^3],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^3(x)\right)\right)\\
&= -\frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^3(x)}\right)}{3b}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^3],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(Expression(Complex(Integer))),fa
iled) cannot be coerced to mode SparseUnivariatePolynomial(Expression(Compl
ex(Integer)))
```

giac [A] time = 0.41, size = 24, normalized size = 0.86

$$-\frac{2\arctan\left(\frac{\sqrt{b\cos(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b*cos(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.12, size = 21, normalized size = 0.75

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cos^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^3)^(1/2),x)
```

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 1.76, size = 39, normalized size = 1.39

$$-\frac{\log\left(\frac{\sqrt{b\cos(x)^3+a}-\sqrt{a}}{\sqrt{b\cos(x)^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*log((sqrt(b*cos(x)^3 + a) - sqrt(a))/(sqrt(b*cos(x)^3 + a) + sqrt(a)))
/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{b\cos(x)^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^3)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**3)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**3), x)
```

3.95 $\int \sqrt{a+b\cos^4(x)}\, \tan(x)\, dx$

Optimal. Leaf size=45

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+b\cos^4(x)}$$

```
[Out] 1/2*arctanh((a+b*cos(x)^4)^(1/2)/a^(1/2))*a^(1/2)-1/2*(a+b*cos(x)^4)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3229, 266, 50, 63, 208}

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+b\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^4]*Tan[x],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]])/2 - Sqrt[a + b*Cos[x]^4]/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\cos^4(x)}\,\tan(x)\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^4(x)\right)\right)$$
$$= -\frac{1}{2}\sqrt{a+b\cos^4(x)} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^4(x)\right)$$
$$= -\frac{1}{2}\sqrt{a+b\cos^4(x)} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^4(x)}\right)}{2b}$$
$$= \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right) - \frac{1}{2}\sqrt{a+b\cos^4(x)}$$

Mathematica [**A**] time = 0.03, size = 45, normalized size = 1.00

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right) - \frac{1}{2}\sqrt{a+b\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^4]*Tan[x],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]])/2 - Sqrt[a + b*Cos[x]^4]/2
```

fricas [**A**] time = 0.91, size = 90, normalized size = 2.00

$$\left[\frac{1}{4}\sqrt{a}\,\log\left(\frac{b\cos(x)^4 + 2\sqrt{b\cos(x)^4+a}\,\sqrt{a} + 2a}{\cos(x)^4}\right) - \frac{1}{2}\sqrt{b\cos(x)^4+a}, -\frac{1}{2}\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos(x)^4+a}\,\sqrt{-a}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log((b*cos(x)^4 + 2*sqrt(b*cos(x)^4 + a)*sqrt(a) + 2*a)/cos(x)
^4) - 1/2*sqrt(b*cos(x)^4 + a), -1/2*sqrt(-a)*arctan(sqrt(b*cos(x)^4 + a)*s
qrt(-a)/a) - 1/2*sqrt(b*cos(x)^4 + a)]
```

giac [**A**] time = 0.47, size = 38, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{\sqrt{b\cos(x)^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} - \frac{1}{2}\sqrt{b\cos(x)^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(sqrt(b*cos(x)^4 + a)/sqrt(-a))/sqrt(-a) - 1/2*sqrt(b*cos(x)^4
 + a)
```

maple [**A**] time = 0.12, size = 44, normalized size = 0.98

$$-\frac{\sqrt{a+b\left(\cos^4(x)\right)}}{2} + \frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^4(x)\right)}}{\cos(x)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^4)^(1/2)*tan(x),x)
```

```
[Out] -1/2*(a+b*cos(x)^4)^(1/2)+1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^4)^(1/2
))/cos(x)^2)
```

maxima [A] time = 2.06, size = 43, normalized size = 0.96

$$\frac{1}{2}\sqrt{a}\,\operatorname{arsinh}\left(-\frac{a}{\sqrt{ab}\left(\sin(x)^2-1\right)}\right)-\frac{1}{2}\sqrt{b\sin(x)^4-2\,b\sin(x)^2+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arcsinh(-a/(sqrt(a*b)*(sin(x)^2 - 1))) - 1/2*sqrt(b*sin(x)^4 -
2*b*sin(x)^2 + a + b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\sqrt{b\cos(x)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos^4(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**4)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**4)*tan(x), x)
```

3.96 $\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] 1/2*arctanh((a+b*cos(x)^4)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3229, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^4],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^4(x)\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^4(x)}\right)}{2b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^4],x]
[Out] ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

fricas [A] time = 0.91, size = 67, normalized size = 2.39

$$\left[\frac{\log\left(\frac{b\cos(x)^4+2\sqrt{b\cos(x)^4+a}\,\sqrt{a}+2\,a}{\cos(x)^4}\right)}{4\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos(x)^4+a}\,\sqrt{-a}}{a}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="fricas")
[Out] [1/4*log((b*cos(x)^4 + 2*sqrt(b*cos(x)^4 + a)*sqrt(a) + 2*a)/cos(x)^4)/sqrt
(a), -1/2*sqrt(-a)*arctan(sqrt(b*cos(x)^4 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.42, size = 24, normalized size = 0.86

$$-\frac{\arctan\left(\frac{\sqrt{b\cos(x)^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="giac")
[Out] -1/2*arctan(sqrt(b*cos(x)^4 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.12, size = 31, normalized size = 1.11

$$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^4(x)\right)}}{\cos(x)^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^4)^(1/2),x)
```

```
[Out] 1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^4)^(1/2))/cos(x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{b\cos(x)^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*cos(x)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{b\cos(x)^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a + b\cos^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**4), x)
```

3.97 $\int \sqrt{a+b\cos^n(x)}\,\tan(x)\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{a+b\cos^n(x)}}{n}$$

```
[Out] 2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n-2*(a+b*cos(x)^n)^(1/2)/n
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{a+b\cos^n(x)}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^n]*Tan[x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/n - (2*Sqrt[a + b*Cos[x]^
n])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos^n(x)}\,\tan(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \cos(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^n(x)\right)}{n}$$
$$= -\frac{2\sqrt{a+b\cos^n(x)}}{n} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^n(x)\right)}{n}$$
$$= -\frac{2\sqrt{a+b\cos^n(x)}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^n(x)}\right)}{bn}$$
$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n} - \frac{2\sqrt{a+b\cos^n(x)}}{n}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.98

$$-\frac{2\sqrt{a+b\cos^n(x)} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^n]*Tan[x],x]
```

```
[Out] -((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cos[x]^n
])/n)
```

fricas [A] time = 0.53, size = 97, normalized size = 2.06

$$\left[\frac{\sqrt{a}\log\left(\frac{b\cos(x)^n+2\sqrt{b\cos(x)^n+a}\sqrt{a}+2a}{\cos(x)^n}\right) - 2\sqrt{b\cos(x)^n+a}}{n}, -\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^n+a}\sqrt{-a}}{a}\right) + \sqrt{b\cos(x)^n+a}\right)}{n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cos(x)^n + 2*sqrt(b*cos(x)^n + a)*sqrt(a) + 2*a)/cos(x)^n)
 - 2*sqrt(b*cos(x)^n + a))/n, -2*(sqrt(-a)*arctan(sqrt(b*cos(x)^n + a)*sqrt
(-a)/a) + sqrt(b*cos(x)^n + a))/n]
```

giac [A] time = 0.56, size = 46, normalized size = 0.98

$$-\frac{2\left(\frac{ab\arctan\left(\frac{\sqrt{b\cos(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\cos(x)^n+a}\,b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -2*(a*b*arctan(sqrt(b*cos(x)^n + a)/sqrt(-a))/sqrt(-a) + sqrt(b*cos(x)^n +
a)*b)/(b*n)
```

maple [A] time = 0.05, size = 39, normalized size = 0.83

$$-\frac{2\sqrt{a+b\left(\cos^{n}(x)\right)}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b(\cos^{n}(x))}}{\sqrt{a}}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^n)^(1/2)*tan(x),x)
```

```
[Out] -1/n*(2*(a+b*cos(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2)
))
```

maxima [A] time = 0.81, size = 58, normalized size = 1.23

$$-\frac{\sqrt{a}\log\left(\frac{\sqrt{b\cos(x)^{n}+a}-\sqrt{a}}{\sqrt{b\cos(x)^{n}+a}+\sqrt{a}}\right)}{n}-\frac{2\sqrt{b\cos(x)^{n}+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*log((sqrt(b*cos(x)^n + a) - sqrt(a))/(sqrt(b*cos(x)^n + a) + sqrt(
a)))/n - 2*sqrt(b*cos(x)^n + a)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\sqrt{a+b\cos(x)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^n)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos^{n}(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**n)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**n)*tan(x), x)
```

3.98 $\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] 2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^n],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \cos(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^n(x)\right)}{n}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^n(x)}\right)}{bn}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^n],x]
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 0.54, size = 74, normalized size = 2.55

$$\left[\frac{\log\left(\frac{b\cos(x)^n+2\sqrt{b\cos(x)^n+a}\sqrt{a}+2a}{\cos(x)^n}\right)}{\sqrt{a}\,n}, -\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*cos(x)^n + 2*sqrt(b*cos(x)^n + a)*sqrt(a) + 2*a)/cos(x)^n)/(sqrt(a)
*n), -2*sqrt(-a)*arctan(sqrt(b*cos(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

giac [A] time = 0.56, size = 27, normalized size = 0.93

$$-\frac{2\arctan\left(\frac{\sqrt{b\cos(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(b*cos(x)^n + a)/sqrt(-a))/(sqrt(-a)*n)
```

maple [A] time = 0.06, size = 24, normalized size = 0.83

$$\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^n)^(1/2),x)
```

```
[Out] 2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [A] time = 1.04, size = 42, normalized size = 1.45

$$-\frac{\log\left(\frac{\sqrt{b\cos(x)^n+a}-\sqrt{a}}{\sqrt{b\cos(x)^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] -log((sqrt(b*cos(x)^n + a) - sqrt(a))/(sqrt(b*cos(x)^n + a) + sqrt(a)))/(sq
rt(a)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(x)}{\sqrt{a+b\cos(x)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^n)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**n)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**n), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```